Rational Institutions Yield Hysteresis

Rafael Di Tella*
Harvard Business School

and

Robert MacCulloch**
London School of Economics

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Abstract

We analyze the government's decision to set unemployment benefits in response to an unemployment shock. The government balances insurance considerations with the tax burden of benefits and the possibility that they introduce adverse "incentive effects" whereby benefits increase the unemployment rate. It is found that when the shock occurs, benefits should be increased in those economies where the adverse incentive effects of benefits are largest. Adjustment costs of changing benefits can introduce hysteresis in benefit setting and unemployment. A good temporary shock can permanently reduce unemployment by making it optimal to have a cut in unemployment benefits. Desirable features of the model are that we obtain an asymmetry out of a symmetric environment and that the mechanism yielding hysteresis is simple (requires that the third derivative of the utility function is non-negative).

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* Morgan Hall, Soldier Field Rd, Boston, MA 02163, USA. e-mail: rditella@hbs.edu. ** STICERD, 10 Furnival Street, London WC2A 2AE, UK. e-mail: robertmacculloch@compuserve.com. We thank Fernando Alvarez for generous ideas and suggestions, Julio Rotemberg for a very helpful conversation and Tim Besley , Juan Pablo Nicolini and participants at various seminars for helpful comments on an earlier draft.

I. Introduction

An important challenge in economics is to build a theoretical model that can explain European unemployment. The ideal model would explain two kinds of asymmetries. The first is an asymmetry over time: it should explain how unemployment could increase after an adverse shock and then remain up for a very long period of time. The second is an asymmetry across countries: ideally the theory should also be able to explain why, once the shock disappears, unemployment drops in some countries but not in others. In this paper we aim to provide one such theory. It is based on the idea that labor market institutions are determined optimally.

The contrasting labor market performances of Europe and the US have been the subject of much research. The standard explanation is based on institutions. It is argued that generous unemployment benefits and strict employment protection drive up European unemployment.[1] Of course this, even if true, would not *explain* either of the two asymmetries. One of the first papers to focus on this problem was Blanchard and Summers (1986). They argued that when wages are set unilaterally by "insiders", wage (rather than employment) gains would follow the withdrawal of a temporary bad shock. The "insider-outsider" model of wage determination (Lindbeck and Snower (1988)) used in these models has been criticized, however, as making a set of special behavioral assumptions about the "insiders" (the debate includes Fehr (1990), Hall (1986), Lindbeck and Snower (1990) and Rotemberg (1999)). Another literature has made the case for plausible asymmetries in the nature of morale and skill decay following joblessness (see Layard and Nickell (1987)). When such "duration" effects are not so severe as to induce withdrawal from the labor force they are a potential source of unemployment persistence. Blanchard and Wolfers (2000), for example, have recently examined the way shocks and exogenous institutions interact to yield unemployment persistence (see also Bertola (1990), Lemieux and McLeod (1998)).

We present a simple model where the government sets the level of taxes on employed workers to pay out unemployment benefits to the unemployed. The economic environment implies that the current rate of unemployment depends on the generosity of

[1] See Bentolila and Bertola (1990), Lazear (1990), Alvarez and Veracierto (1999), Caballero and Hammour (1998), *inter alia*. See Gregg and Manning (1996) for a review.

unemployment benefits and a shock. A key feature of our model is that, for some simple cases, we can evaluate the effects of an increase in the level of risk in the economy. Since unemployment benefits are supposed to provide insurance, the level of risk is a key parameter in the formulation of the problem. A large literature in public economics examines the optimal provision of unemployment insurance. Important papers include Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) on how UI benefits ought to be paid over time, Feldstein (1978) and Topel (1983) on the effect of UI (and UI-financing) on layoff and quit behavior and Mortensen (1977) on the effect on job search. In general, however, this literature does not look at the problem of providing unemployment insurance when the level of risk in the environment changes. Changing these models to address these questions is not always feasible. For example, the problem studied by Hopenhayn and Nicolini (1997) is how to achieve a certain level of insurance at minimum cost, so that changing some risk parameters in the problem will not answer the questions we are after.[2] A previous paper, Di Tella and MacCulloch (2000), shows a simpler version of the present model (without shocks) and presents some comparisons across steady states. It also presents evidence consistent with the idea that unemployment benefits tend to fall with the unemployment rate (a tax effect) and increase when there are positive changes in the unemployment rate (an insurance effect).[3]

In an important review article, Blanchard and Katz (1997) have suggested that if unemployment shocks lead to increases in unemployment benefits, then we may have a way to explain the high persistence of European unemployment. In this paper we formalize their intuition, and show how such "endogenous institutions" lead to hysteresis, even in a *normative* model of unemployment benefits. Our first task in the paper is to formalize the

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[2] Hansen and Imrohoroglu (1992) present a model showing how costly it is to set the wrong (non-optimal) level of unemployment benefits in a general equilibrium model where there are liquidity constraints and moral hazard. We experimented with a (much) simpler version of that model to see if it could be used to study the determination of unemployment benefits at different levels of risk. The fundamental problem encountered is that the parameters that determine the unemployment rate and that could be used to capture the risk in the environment *also* affect the degree of risk aversion that individuals have. Thus, it is impossible to disentangle in that model what is happening because individuals have become more risk-averse and what occurs because the environment is more risky.

[3] The general structure is related to the voting model of Wright (1986) (see also Atkinson (1990)). Neither of these models, however, considers the role of "incentive effects". These can be thought of as the coefficient on benefits in an unemployment regression. Saint Paul (1996) presents a good review of positive models, and discusses other institutions (such as job security provisions).

meaning of hysteresis. We use a pay-off function, $S(b,e)$, where b is a choice variable and e is a stationary random variable whose value is known when b is set. Changes in the value of e (for example from 0 to $e1$) correspond to shocks. Put simply, hysteresis exists when the value of adjusting the choice variable, b, once the shock occurs, DS^{e1}, is bigger than the value of adjusting back once the shock has disappeared, DS^{0}. This is true as long as there is some "adjustment cost" that lies *strictly between these two values*. Note that unless strong restrictions are placed on the functional form of $S(b,e)$ to guarantee the occurrence of the special case in which $DS^{0}=DS^{e1}$, hysteresis as we define it here can be a pervasive feature of the world. Provided the adjustment cost has the "correct" size, a sufficient condition for hysteresis to exist for positive shocks is that the degree of concavity of *S(.)* rises with the size of the shock, making the pay-off function more sharply "peaked". Conversely, if the degree of concavity of *S(.)* falls with the size of the shock, making the pay-off function less sharply "peaked", then hysteresis can exist for negative shocks.

This may be helpful in putting more structure to the definition of hysteresis, but as such, says little about European unemployment. For all we know, a formulation where economic variables are used to construct *S(.)* could lead to all the wrong correlations. For instance, it could be that a shock that increases unemployment leads to lower unemployment benefits and higher social welfare. This would hardly be descriptive of the European experience since the 1970's. The challenge for the second part of the paper is to show how, when *S(.)* is a reasonable social welfare function such as the weighted sum of the utility of the employed and the unemployed, a shock that increases unemployment can reduce social welfare and lead to permanently higher levels of benefits and unemployment even after the shock has disappeared. This will happen if two things occur: the degree of concavity of *S(.)* increases once the shock occurs and the shock leads to a higher optimal level of unemployment benefits.

A key condition for the degree of concavity of *S(.)* to increase with the shock is that the individual utility function has a non-negative third derivative. In other words, we require that individuals do not become more risk averse at higher income, a condition that is satisfied by most utility functions commonly used (for example, logarithmic, CRRA, CARA). The reason why this leads to hysteresis is because all of the effects of the shock on the concavity of *S(.)* have the same sign. When an unemployment shock takes place, the new social welfare function incorporates some people on benefits and loses an equal number of people on wages. As long as the replacement rate is less than one, the change consequently incorporates people who are on a more concave part of their utility function. A second effect is that higher benefit payments to the unemployed mean a higher tax burden. This means lower net wages, so that now the employed are also on a more concave part of the utility function.[4]

The second condition for European-style hysteresis to exist, namely that unemployment benefits ought to be *increased* following an unemployment shock, is that the adverse incentive effects of benefits are large. The larger these incentive effects are, the more likely it is that the optimal response to a shock is to raise benefits. The intuition is simple once we note that benefits are set optimally at all times, including the moment just before the shock takes place. If incentive effects are large, benefits ought to be set low prior to the shock to minimize unemployment problems generated by the welfare state. In the limit, we can imagine a situation where unemployment is zero if benefits are zero. Then it is clear that the optimal level of benefits prior to the shock must be zero. But after the shock takes place, the marginal gain from an extra unit of insurance is particularly large.

In contrast to previous models in the literature, the mechanism that yields hysteresis is no longer relevant when unemployment becomes large because tax considerations provide a self-correcting mechanism (see Hall (1986)). Furthermore, it is simple (requires that the third derivative of the utility function is non-negative) and symmetric in the sense that the same mechanism is at play in the presence of negative and positive shocks. In particular, it does not assume any behavioral asymmetry, between insiders or outsiders or between the long-term and the short-term unemployed. The only ad-hoc feature is that it requires the existence of "adjustment costs" that are not modeled. We conjecture that an in-depth look at the extended benefit system in the US may provide some empirical clues as to what may give rise to these costs.

Although the paper deals with unemployment insurance, the results seem to have a more general application to other situations where the objective function depends on an individual's utility function. Two key features of this paper - the fact that a shock increases the concavity of the objective function and there are some adjustment costs - are present in

[4] Another effect with the same sign is described after Proposition 4.

the rational design of other institutions such as job security provisions or minimum wages. Section II provides a definition and a brief outline of how rational hysteresis can be generated. Section III presents the general problem in a simple economic model of optimal benefit setting and solves the simplest case when there is full discounting and no adjustment costs to develop the basic intuition. Section IV includes the effect of an adjustment cost of changing the benefit level and derives the conditions required for hysteresis to occur. Section V studies the consequences of good and (very) bad shocks and the implications for the definition of the natural rate of unemployment. Section VI concludes.

II. Formal Definition of Hysteresis

Define a function $S(b,e)$ where $¶^2S(b,e)/¶b^2<0$. Assume e is a random shock, $b^0=argmax_b$ $S(b,0)$ and $b^{e1}=argmax_b$ $S(b,e1)$. Without loss of generality, assume $b^0<b^{e1}$. Assume there is a fixed cost, m, of changing b. Each period b is set to maximize $S(b,e)$, less the adjustment cost, after observing the value of e. The gain derived from adjusting from b^0 to b^{e1} equals DS^{e1}-$m=S(b^{e1},e1)-S(b^0,e1)-m$ and the gain from adjusting back from b^{e1} to b^0 equals DS^0-$m=S(b^0,0)$-$S(b^{e1},0)-m$.

Proposition 1:

Assume that $min(DS^0,DS^{e1})<m< max(DS^0,DS^{e1})$.

a. If $\frac{\partial^2 S(b^{e1}-x,e1)}{\partial b^2}<\frac{\partial^2 S(b^0+x,0)}{\partial b^2}<0 \ \ \forall x\in(0,b^{e1}-b^0)$ then hysteresis exists for a shock of size, $e1$.

b. If $0>\frac{\partial^2 S(b^{e1}-x,e1)}{\partial b^2}>\frac{\partial^2 S(b^0+x,0)}{\partial b^2} \ \ \forall x\in(0,b^{e1}-b^0)$ then hysteresis exists for a shock of size,$-e1$.

Proof: See Appendix I. #

Figure 1 illustrates. Proposition 1 starts by assuming a specific value of adjustment costs. It then argues, rather trivially, that when a shock occurs the original function has to become, in some precise sense, either more "concave" or less "concave". Part (a) of Proposition 1 refers to the first case and proves that there is hysteresis for positive values of the shock. Part (b) refers to the second case, where the degree of concavity of the objective function falls in absolute size over the region (b^0, b^{e1}) due to a shock, and proves that hysteresis will exist for negative values of the shock. Note that, because the condition for hysteresis involves comparing concavity of two functions at two different values of the choice variable, a condition on the change in concavity at a given point is enough only in the cases where $|b^{e1}-b^0|$ is sufficiently small. What is required in the general case is a comparison of concavity of the original function around its maximum and concavity of the new function around the new maximum. This is what the terms b^0+x and $b^{e1}-x$ refer to. Less formally, hysteresis exists when the objective function becomes more sharply "peaked" in the presence of the shock. If the second derivative remains constant for all values of *b*, which would be the case for an objective function of the form, $Q(b)+eb$, where $Q(b)$ is a quadratic function of *b*, then there can be no hysteresis.

The conditions required in Proposition 1 could be satisfied by a potentially large number of functional forms. However we still must check that it can hold for a social welfare function: $S(b,e)=SW(b,u,T,e)$ where *b* is the level of unemployment benefits, $u=f(b,e)$ is the unemployment rate and $T=g(b,u)$ is the level of taxes. More importantly, once economic relationships are considered, nothing leads us to expect that hysteresis will occur with the correct co-variation in the variables. For example, a shock that increases unemployment could easily lead to a lower level of optimal benefits. In other words, nothing precludes that in Figure 1 the function with the shock, *S¢S¢*, has a maximum to the left of b^0. Put differently, we ask if the predictions in our model are compatible with European unemployment, where we ask whether a shock that subsequently disappears can leave benefits at a higher level, unemployment at a higher level and social welfare at a lower level in future periods, compared to their past values. This is the challenge for sections III and IV of the paper.

III. A Simple Model of Unemployment Benefit Determination

III. A. Individual Preferences

Assume an economy populated with identical risk-averse individuals with strictly concave utility defined over income, $U(i)$ (where *U¢(i)>0* and *U²(i)<0*). Individuals cannot save or insure themselves in private insurance markets.[5] The unemployment benefit program pays b_t to the unemployed, funding it with a tax equal to T_t levied on employed individuals at time, t.

III. B. Labor Market

At any point in time we denote the equilibrium unemployment rate, $u_t=f(b_t,\varepsilon_t)$, where ε_t is a random, stationary shock defined on $[\varepsilon^l,\varepsilon^h]$. It has mean zero and $\partial u_t/\partial\varepsilon_t>0$. Unemployment is also affected by the generosity of the unemployment benefit program, b_t, where $\partial u_t/\partial b_t>0$.

This is the equilibrium, for example, in the following simple model of an economy.[6] Assume that firms pay workers the gross real wage, Y_t^g, and competition ensures zero profits: $\pi(Y_t^g,\varepsilon_t)=0$ (where $\partial\pi/\partial Y_t^g<0$, $\partial\pi/\partial\varepsilon_t<0$). Assume workers can shirk on their job (in which case their work effort equals 0) but if caught, they are fired. The expected income from being fired equals the probability of staying unemployed ($=a(X_t)$ where X_t is the unemployment rate and $\partial a/\partial X_t>0$) multiplied by the level of benefits, plus the probability of finding a new job ($=1-a(X_t)$) multiplied by the wage net of taxes and effort costs. Where we are assuming that newly hired workers who have been caught shirking once are not able to shirk again. The "No-Shirking-Condition" equates the value from exerting effort on the job to the value of shirking: $C(Y_t^g,b_t,X_t)=0$ (where $\partial C/\partial Y_t^g>0$, $\partial C/\partial b_t<0$, $\partial C/\partial X_t>0$). Equilibrium unemployment, u_t, can now be expressed as a function of both the level of benefits and the shock ($u_t=f(b_t,\varepsilon_t)$ where $\partial f/\partial b_t>0$, $\partial f/\partial\varepsilon_t>0$). The equilibrium gross wage can be expressed solely as a function of the shock ($w_t^g=w^g(\varepsilon_t)$ where $\partial w^g/\partial\varepsilon_t<0$).

III. C. The Government's Problem

We assume that the shock occurring at time t is random but known when benefits are set at time t.[7] There is an adjustment cost, m_t, to changing the level of the policy variable, unemployment benefits. This could be due to several factors, including administrative costs

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[5] On the role of private information in explaining the failure of private insurance markets, see Chiu and Karni (1998).

[6] This can be derived from a variety of standard models of equilibrium unemployment, including an efficiency wage model, a union bargaining model or a search model.

[7] The assumption about the timing ensures that, for the cases we analyze, the level of unemployment at any point in time is the relevant measure of "risk" in the economy. Other timings would require us to look at the distribution of ε.

and the costs of coordination that are incurred if political support for such changes is required. The government must pay the same cost both when it wants to increase benefits and when it wishes to cut them. Clearly, allowing for differences in adjustment costs would make it more likely that hysteresis obtains

After observing the shock, the government's problem is to set benefits to maximize the present discounted value of expected welfare, conditional on information at time t, subject to the budget constraint, the possibility that higher benefits may cause higher unemployment and the adjustment costs. If the social rate of time preference equals q, the government's problem as of time zero is:

$$\max_{b_0,\, b_1,\, \ldots} \; SW_0 \; + \; \mathrm{E}\left[\sum_{t=1}^{\infty} \frac{SW_t - M_t}{(1+q)^t} \;\middle|\; t=0\right] \tag{1}$$

subject to

$$u_t = f(b_t, e_t) \qquad \text{Incentive Constraint} \tag{2}$$

$$T_t = \frac{u_t b_t}{1-u_t} \qquad \text{Budget Constraint} \tag{3}$$

$$M_t = \begin{cases} m_t \geq 0 & \text{if } \; |b_t - b_{t-1}| \neq 0 \\ 0 & \text{if } \; |b_t - b_{t-1}| = 0 \end{cases} \qquad \text{Adjustment Costs} \tag{4}$$

where $SW_t=(1-u_t)U(w_t)+u_tU(b_t)$ and $w_t=w^g(e_t)-T_t$ is the net wage.[8] Substituting in SW_t for constraints (2) and (3) yields $S(b_t,e_t)$. This formulation implies the simplest assumption regarding transitional dynamics: each period the government ignores the employment history. Thus, a situation where a person is unemployed for two periods is identical to situation where that person is unemployed one period and another is unemployed the next. If we define the value function as:

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[8] Alternatively, the same social welfare function (divided by the discount rate) is obtained if we consider the lifetime expected utility of employed and unemployed workers used in Shapiro and Stiglitz (1984). Transitional dynamics are analyzed in Kimball (1994).

$$V(b_{t-1}, e_t) = \max_{b_t,\ b_{t+1},\ \dots} \ \mathrm{E}\left[\sum\nolimits_{s=t}^{\infty} \frac{S(b_s, e_s) - M_s}{(1+q)^{s-t}} \ | \ t\right] \tag{5}$$

then the solution to the government's problem satisfies:

$$V(b_{t-1}, e_t) = \max_{b_t} \ \{S(b_t, e_t) - M_t + (1+q)^{-1} E[V(b_t, e_{t+1}) \,|\, t]\} \tag{6}$$

This Bellman equation fully characterizes the solution to the government's unemployment benefit problem. More intuition can be gained, however, by examining the government's problem in extreme cases, such as when there is full discounting or when adjustment costs are zero.

III. D. Basic Results with Full Discounting and No Adjustment Costs

As is standard in this type of problem, it is useful to start by assuming that only current period welfare is valued and there are no adjustment costs. The problem reduces to:

$$\max_b \qquad SW(b, u, T, e) = (1-u)U(w^g(e) - T) + uU(b) \tag{7}$$

subject to $u = f(b, e)$ Incentive Constraint (8)

$T = \dfrac{u\,b}{1-u}$ Budget Constraint (9)

The First Order Condition (FOC) is:

$$-(1-u)U'(w)[\frac{u}{1-u} + \frac{b}{(1-u)^2}\frac{\partial u}{\partial b}] + u\,U'(b) - \frac{\partial u}{\partial b}[U(w) - U(b)] = 0 \tag{10}$$

When the second order condition holds, the FOC implicitly defines optimal benefits as a function of the magnitude of the incentive effects, $\partial u/\partial b$. Clearly if there are no adverse incentive effects of benefits, marginal utility must be equalized across states and we simply have full insurance. Inspection of the FOC above suggests that incentive effects would sometimes tend to reduce the optimal level of benefits. For simplicity, we assume that incentive effects are linear and that the shock is additive.[9] At each point in time, unemployment is given by $u=u^f+\alpha b+\varepsilon$.[10] This equals the sum of frictional unemployment, u^f, unemployment arising from the adverse incentive effects of the benefit system, αb, and from random shocks, ε.

Proposition 2: The government should set benefits low when incentive effects are large.

Proof: Compute $db/d\alpha<0$, using the implicit function rule on the FOC (10). #

The intuition for this result is simple. At the optimum, the government balances insurance against tax costs to fund the program and the adverse incentive effects that unemployment benefits introduce, which increase unemployment. When incentive effects are large, the government will try to restrict benefits because, for a given level of insurance, benefits now have a bigger effect on the unemployment rate and the tax burden of the employed.

We can now study what happens to the optimal level of benefits when there is an exogenous shock to the unemployment rate.

Proposition 3:

a. When incentive effects are small, the government should reduce benefits following the occurrence of an adverse shock.
b. When incentive effects are large, the government should increase benefits following the occurrence of an adverse shock.

Proof: See Appendix I. #

If there are only small incentive effects of benefits on unemployment, benefits should

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[9] A sufficient condition for the Second Order Condition to hold under these conditions is $\alpha \le u^f$. It is possible to derive some of the results below for other cases as well, available on request.

[10] This makes two simplifying assumptions. First, a linear approximation has been used. Second, we are implicitly assuming that the shock does not directly affect "incentives" in the labor market, or $\partial^2 f/\partial\varepsilon\partial b=0$. The reason is tractability. For details, as well as empirical evidence on the determination of benefits, see Di Tella and MacCulloch (1995).

decrease due to exogenous adverse shocks to unemployment. The reason is that benefits should be initially set at relatively generous levels (the replacement ratio is close to 1) when α is small, and the main impact of the shock is then to raise taxes (via the budget constraint) and reduce the affordable level of benefits.

Perhaps the more interesting case is when incentive effects are large. Initially, unemployment insurance is set at relatively low levels and the optimal response to an adverse unemployment shock may be to *increase*, rather than reduce, the generosity of unemployment benefits. In such a case increases in insurance have large positive marginal effects on social welfare in the presence of an adverse shock. Consider an example where utility is logarithmic. If $U(x)=\log x$ then pre-shock social welfare is $S(b,0)=u\log b+(1-u)\log[w^g-ub/(1-u)]$ where $u=u^f+\alpha b$. This can be re-expressed as $S(b,0)=\log w^g+u\log(b/w^g)+(1-u)\log[1-u(b/w^g)/(1-u)]$. For simplicity, let $\partial w^g/\partial\varepsilon=0$. When benefits are set low so that $b<<w^g$, taxes are low and consequently $S(b,0)\approx\log w^g+u(\log(b/w^g)-b/w^g)$ (since $\log(1+x)\approx x$ for small x). In the presence of a shock that adds $\varepsilon 1$ to the unemployment rate, $S(b,\varepsilon 1)\approx S(b,0)+\varepsilon 1(\log(b/w^g)-b/w^g)$. The second term has a positive derivative with respect to b, equal to $\varepsilon 1(1/b-1/w^g)$. Consequently if benefits were being set optimally before the shock occurred, well below the wage due to the large incentive effects, there now exists a positive marginal welfare gain from more insurance. The smaller is the initial level of benefits, the larger is the gain from adjusting.

A fundamental aspect of this problem is that the effect of an adverse shock on the objective function (social welfare) is to increase its degree of concavity for a given value of benefits. In other words, the second derivative of the welfare function, with respect to benefits, becomes more negative in the presence of the shock.

Proposition 4: Provided $U'''(w)\geq 0$ then

$$\frac{\partial^2 S(b,\varepsilon 1)}{\partial b^2} < \frac{\partial^2 S(b,0)}{\partial b^2} \quad \forall b,\ \ \forall \varepsilon 1>0 \tag{11}$$

Proof: See Appendix I. #

There are several effects that give rise to this result. First, an adverse shock shifts a proportion of workers from employment to unemployment. Once unemployed they find themselves on a lower part of their utility function (where U'' is more negative) since they are now only earning the benefit (which is lower than the wage). Second, an adverse shock cuts the level of net wages by lowering the gross wage that workers are paid and by increasing the level of taxes due to the greater numbers of unemployed. Hence even those workers who stay employed are pushed onto a lower part of their utility function (where U'' is more negative). Third, the greater numbers of unemployed due to the shock mean that higher benefits have increasingly more severe effects on taxes, which also makes the second derivative of the welfare function more negative.

For the logarithmic utility example where incentive effects are large so benefits are initially set low, the expression for $\partial^3 S/\partial e \partial b^2$ is dominated by the negative term, $-1/b^2$. This term captures the degree of concavity of the utility function, $U(b)=\log b$, of the workers who are made unemployed due to the shock. More generally, Figure 1 in Appendix I shows the case when incentive effects are large. Social welfare varies with benefits along the curve *SS* in the absence of a shock. The optimal level of benefits is set relatively low at b^0. Social welfare is $S(b^0,0)$ at point A. This figure also shows the impact of a shock to unemployment, $e1>0$. Social welfare now varies with benefits along the curve $S'S'$. From Proposition 3(b) we know that the optimal level of benefits rises to b^{e1} and social welfare equals $S(b^e,e1)$ at point C. From Proposition 4 we know that, for a given b, the degree of concavity of the post-shock welfare function, $S'S'$, is greater than the degree of concavity of the pre-shock function, *SS*.

III. E. Results Without Full Discounting and No Adjustment Costs

Assume that the government positively weights welfare in future periods and the adjustment cost is zero. The solution to problem (1) remains the same as in sub-section *III.D* since benefits should be set each period at the level that maximizes $S(b_t,e_t)$.

IV. Optimal Benefits with Positive Adjustment Costs Yield Hysteresis

In sub-sections *IV.A* and *IV.B* we assume that there exists a positive fixed cost of adjusting benefits. In other words, $m_t=m>0$ if $|b_t-b_{t-1}|\neq 0$ and is zero otherwise. Sub-section *IV.C* discusses empirical implications.

IV. A. The Case with Positive Fixed Costs of Adjustment

If the fixed cost of adjusting benefits is very large so benefits must be set initially at a single level that cannot be changed in future periods, then the optimal level of benefits solves the problem: $max_b\ S(b,e_0) + E[S_{t=1}^{¥}\ S(b,e_t)/(1+q)^t \mid t=1]$.

For intermediate levels of adjustment costs, hysteresis in benefit setting can arise. The easiest way to see this is to start from an equilibrium where $e=0$ and benefits are being set to maximize social welfare in the current period. Let $b^0=argmax_b\ S(b,0)$, $b^{e1}=argmax_b\ S(b,e1)$, $DS^0=S(b^0,0)-S(b^{e1},0)$ and $DS^{e1}=S(b^{e1},e1)-S(b^0,e1)$.

Proposition 5 (Hysteresis): Consider the effect of an adverse shock to unemployment of size, $e1>0$. For sufficiently high rates of time preference, hysteresis can exist for a range of adjustment costs.

Proof: See Appendix I. #

The intuition is that social welfare, drawn as a function of benefits, becomes more sharply "peaked" in the presence of an adverse shock to unemployment. Consequently there exists an asymmetry between the welfare gain from adjusting benefits in the presence of the shock and the welfare gain from adjusting benefits once the shock has gone. Note that our results do not depend on q®¥. Although the general problem is quite complex, we can gain some understanding about the behavior of the problem by considering an extreme case. Assume that a bad shock hits the economy and it is known, ex-ante, that the shock will disappear after one period and that it will never return. It is easy to see that unemployment benefits should be adjusted provided $DS^{e1}-m>DS^0/q$. After benefits have been changed and the shock has disappeared forever (i.e. $e=0$ for all current and future t) the government may still want to keep benefits at their old level. By not changing the government saves on adjustment costs today, but loses the social welfare gain of having the "correct" level of benefits in the future. This means that there will be hysteresis provided $m>DS^0(1+q)/q$. Thus, even if the shock takes an extreme form, and the future is not completely discounted, there will be hysteresis as long as:

$$\Delta S^{e1} - \Delta S^{0}/q > m > \Delta S^{0}(1+q)/q \qquad (12)$$

Note, however, that as the rate of time preference becomes small there can be no hysteresis. When there exists such hysteresis in benefit-setting, there exists a corresponding hysteresis effect in unemployment. The reason is that if unemployment benefits are increased in the presence of a temporary shock but not subsequently reduced, then the unemployment rate will also increase but not subsequently return to its pre-shock level. The extent of the rise in unemployment will depend on the size of the incentive effects of benefits.

Figure 1 illustrates. Benefits are set at the pre-shock level, b^0, where social welfare equals $S(b^0,0)$ at point A. In the presence of an adverse shock to unemployment, *e1*, social welfare now varies with benefits along the curve *S¢S¢*. If benefits are kept at their pre-shock level then welfare drops to $S(b^0,e1)$ at point B. However if benefits are increased to b^{e1}, which maximizes *S'S'*, then social welfare can be increased to $S(b^{e1},e1)$ at point C. In other words, before paying adjustment costs, increasing benefits after the shock raises social welfare by DS^{e1}. After the shock disappears, welfare equals $S(b^{e1},0)$ at point D. The gain from reducing benefits from b^{e1} to the optimal value b^0 (their initial value) equals DS^0 before paying the adjustment cost. The welfare gain from increasing benefits in the presence of the shock, DS^{e1}, is larger than the size of the welfare gain from reducing benefits after it has gone, DS^0. If adjustment costs are zero (or small) then along the optimal path benefits should be increased from b^0 to b^{e1} and then subsequently reduced back to b^0. However if adjustment costs are larger, satisfying $DS^0<m<DS^{e1}$, and the government's rate of time preference is high so that it only values current period welfare, then it is worthwhile for benefits to be increased to b^{e1} in the period that the shock occurs but not reduced once it disappears. The unemployment rate also does not return to its initial value, due to the higher level of benefits.

IV. B. (S,s) Rules for Institutions and a Measure of the Degree of Hysteresis[11]

We start by characterizing the amount of hysteresis in the economy.

Definition 1: Let r and e^* be two shocks such that:

$$S(b(-r),-r) - S(b(0),-r) = m = S(b(e^*),e^*) - S(b(0),e^*) \tag{13}$$

then the degree of hysteresis in the economy, h, is characterized by:

$$h = \left| \frac{b(-r) - b(0)}{b(e^*) - b(0)} \right| \tag{14}$$

Given the uncertainty structure, this measure best captures the asymmetric range of inaction of the government when it sets benefits. When h is larger than one, it reflects the asymmetry resulting from the increase in the degree of concavity of the social welfare function in the presence of an unemployment shock. The more the degree of concavity rises, the larger h becomes. The more concavity does not change after a shock, the closer h is to one.

The nature of this measure can be seen in Figures 2 and 3. They are drawn for the case with large incentive effects and when only current period welfare is valued due to a high rate of time preference. Figure 2 shows two functions, both of which define social welfare (in the absence of adjustment costs) as a function of the size of the unemployment shock, e. The function, $S(b(e),e)$, which is depicted by the thick line, shows how social welfare varies with e when benefits, $b^e=b(e)$, vary optimally so as to maximize $S(.)$ for each level of e. In other words, if $b(e)=argmax_b\, S(b,e)$ then:

$$\frac{dS(b(e),e)}{de} = \frac{\partial S(b,e)}{\partial b}\frac{\partial b}{\partial e} + \frac{\partial S(b,e)}{\partial e} = \frac{\partial S(b,e)}{\partial e} \tag{15}$$

by the Envelope Theorem. The function, $S(b(0),e)$, which is depicted by the thin line, shows how $S(.)$ varies with e when benefits are fixed at the level, $b^0=b(0)$, where $b(0)=argmax_b\, S(b,0)$.

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[11] We thank Fernando Alvarez for ideas and help with this section. Errors are our own.

These two functions are tangential when $e=0$. For other values of the shock, $S(b(0),e)<S(b(e),e)$. If benefits are set initially at b^0, then the increase in welfare that can be obtained from changing the level of benefits when there is an adjustment cost of size, m, equals $S(b(e),e)-S(b(0),e)-m$. This is the motivation for the proposed definition of h.

Figure 3 draws the same problem, but in (e,b) space. The thick line, $b(e)$, describes how benefits vary optimally so as to maximize $S(.)$ for each level of the shock, e. It is upward sloping since we are focusing on the case where incentive effects of benefits are large (see Proposition 3(b)). The thin lines depict the limits of the regions of inaction. In the absence of a shock, benefits are set optimally at $b^0=b(0)$. In the presence of an adverse shock to unemployment smaller than e^* benefits should not be changed due to the cost, m, of doing so. In the presence of a shock that reduces unemployment, benefits should also not be changed provided that $e>-r$. In this figure, h is the vertical distance between points D and F divided by the vertical distance between points A and B. If there are no changes in the degree of concavity of $S(b,e)$ along FB then these two distances must be the same.

Consider the example of a shock that is marginally larger than e^*. Benefits should be increased from $b(0)$ to $b(e^*)$ (from point A to point B in Figure 3). Once the shock has disappeared, benefits should be kept at $b(e^*)$. Only if a shock reduces unemployment by more than the level measured by the horizontal distance between points O and C should benefits be cut. Figures 2 and 3 show that the nature of the solution to problem (1) follows an (S,s) rule: when the size of the shock deviates sufficiently from its previous value at which benefits were being set optimally, benefits should be adjusted so that they become optimal for the size of the new shock.

Further intuition can be gained by expressing the degree of hysteresis in terms of the concavity of the objective function. Provided $|b(e^*)-b(-r)|$ is small, then $m \approx -\frac{1}{2}\frac{\partial^2 S(b(-r),-r)}{\partial b^2}[b(-r)-b(0)]^2$ and $m \approx -\frac{1}{2}\frac{\partial^2 S(b(e^*),e^*)}{\partial b^2}[b(e^*)-b(0)]^2$ (using 2nd order Taylor approximations). Hence:

$$h = \left|\frac{b(-r)-b(0)}{b(e^*)-b(0)}\right| \approx \sqrt{\frac{\partial^2 S(b(e^*),e^*)}{\partial b^2} / \frac{\partial^2 S(b(-r),-r)}{\partial b^2}} = \sqrt{(1+\Delta C)} \qquad (16)$$

where *DC* is the percentage change in concavity. In other words, our measure of hysteresis suggests that the ratio of the ranges of inaction is proportional to the square root of one plus the percentage change in concavity.

IV. C. Discussion and Empirical Implications

Figure 4 summarizes the workings of the model. The thick line in the northeast quadrant shows how the level of social welfare (in the absence of adjustment costs) varies with the level of benefits as the value of the shock changes and benefits are adjusted optimally. Bigger adverse shocks lead to movements down the thick line from points A to C as welfare decreases and the optimal level of benefits first rise and then falls. If benefits were not adjusted optimally then the drop in welfare would have been greater, as depicted by the thin lines drawn under the envelope in the northwest quadrant. The line in the southwest quadrant shows optimal benefits, b^e, as a function of the shock plus the level of frictional unemployment. Clearly, if $e+u^f=0$ then $b^e=0$. As the size of the adverse shock increases, b^e rises before ultimately falling as tax effects begin to dominate benefit setting. Again, it is clear that when $e+u^f=1$ then benefits should be zero. In the presence of adjustment costs, zones of inaction are depicted around points D and B. The smaller zone around point B compared to point D reflects the greater degree of concavity of the welfare function as the size of the adverse shock increases. Since the zone of inaction around B does not include b^0, benefits should be raised to b^{e1} when the economy is hit by a shock of size, $e1$. Once the shock disappears, the zone of inaction around D includes b^{e1}. Hence no changes in benefits ought to take place.

Our results suggest that European unemployment, in some limited sense, may be optimal.[12] If there were high costs of changing institutions in Europe, the optimal course of action after the oil shocks in the 70's could well have been to increase benefits and not lower them after the oil price came down. If benefits increase unemployment, this would lead to a period of protracted unemployment. The differential performance between the US and Europe (see Figures 5 and 6 for a comparison with Spain) could be explained if the cost of

[12] See section V.A. below for an elaboration on the optimality of high unemployment.

changing institutions is lower in the US.[13] The implication is that the coordination, legislative and political costs of changing the level of benefits would be higher in Europe.

A particularly simple way of doing this would be to specify explicitly in advance a rule or formula defining how benefits are going to be adjusted in the presence of a shock. An example would be a contingent rule such as unemployment benefits are x if the unemployment rate is less than y and z if it rises above y. The U.S. has such a rule in the Extended Compensation Act. Japan and Canada also have variants of these laws. These countries have laws *stating* that benefits depend on aggregate unemployment conditions. In the U.S., the Federal/State Extended Compensation Act of 1970 established a second layer of benefits for claimants who exhaust their regular entitlement during periods of relatively high unemployment in a State. This program provides for up to 13 extra weeks of benefits at the claimant's usual weekly benefit amount. The benefits are triggered on "*if the State's insured unemployment rate for the past 13-week period is 20 percent higher than the rate for the corresponding period in the past two years and the rate is at least 5 percent.*" Hence in the U.S. adverse shocks that increase the unemployment rate also increase benefit generosity, by law. If this type of legislation lowers the adjustment costs of changing benefits we may expect to observe less evidence of hysteresis: benefits would be more likely to be increased in the presence of an adverse shock but returned to their initial value once the shock has gone. In fact, the U.S. Federal/State Extended Compensation Act does specify that "*extended benefits cease to become available when the insured unemployment rate does not meet either the 20 percent requirement or the 5 percent requirement*". In other words, the primitive in our model is differences in how easy it is change unemployment benefits rather than differences in the generosity or otherwise of these programs across countries. In some sense this is closer to the way the word "institution" is often used by authors such as North (1990), where attention is given to the rules of the game, rather than the outcomes.

From a positive point of view, an empirical prediction of the model is that, other things equal, differences in the government's rate of time preference should explain changes in the unemployment benefit system. A possible way of capturing differences in impatience

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[13] The OECD measure of benefits describes the parameters of the unemployment benefit system. It is calculated as the pre-tax average of the unemployment benefit replacement ratios for two earnings levels, three family situations and three durations of unemployment (see the OECD Jobs Study (1994) for details).

is to focus on political color, as it is sometimes argued that left-wing parties discount the future more than right-wing parties. It is also possible that the length of the electoral cycle influences the government's discount rate. Then we may expect to see less evidence of benefit hysteresis in countries that have longer periods between elections.

V. Good Shocks, (Very) Bad Shocks and the Natural Rate

In this section, we contrast the predictions of our model with those from previous hysteresis models.

V. A. An Example with a Good Shock

A standard prediction in previous models is that if good shocks can permanently reduce unemployment, then some role for an active government policy can be justified. Of course, such policies may introduce other costs (e.g. in terms of inflation). The same is true in our model, although for somewhat different reasons. The key effect of a good shock is that it *temporarily lowers unemployment risk in the economy*. When this occurs, there is less demand for insurance and a relatively large welfare gain to be captured by cutting benefits. Provided this gain now exceeds the adjustment cost of changing benefits (and the other costs associated with the shock such as inflation, unpredictability, etc) the government can increase social welfare by reducing benefits and keeping them low in future periods (in the absence of further shocks).

In Figure 7 benefits are initially set equal to b^0 and welfare is at point A. Assume that the government's rate of time preference is high and that there exists an adjustment cost, $m=DS^0+c<DS^{e1}$ (where c is a small cost). In the presence of an adverse shock to unemployment, benefits are increased to b^{e1} and social welfare (before paying the adjustment cost) equals $S(b^{e1},e1)$ at point C. Once the shock disappears, and in the absence of further shocks, benefits remain at b^{e1} and social welfare equals $S(b^{e1},0)$ at point D.

Now consider the effect of a good shock that lowers the rate of unemployment by q, but has costs, K, associated. This shock could certainly be a monetary shock, and K

represents the costs of inflation.[14] The direct effect of the shock is to increase social welfare to $S(b^{e1},-q)$ at point E on the curve, *S¢¢S¢¢*. Clearly if the welfare gain from the good shock is less than zero, $S(b^{e1},-q)-S(b^{e1},0)-K=m<0$, the government should avoid this shock. In traditional models, with exogenous benefits fixed at b^{e1}, the good shock could only lead to a *temporary* reduction in unemployment and a (possibly permanent) increase in inflation. Thus, only in very peculiar cases could such an action be justified.

However if benefits are set optimally they should be cut to b^{-q} in the presence of the good shock so that welfare can be further increased by $DS^{-q}-m$ (at point F). Provided $DS^{-q}-m+m>0$ the shock has a positive overall effect on welfare once benefits are endogenized. After the shock has disappeared (and in the absence of further ones) the level of benefits remains low and consequently the unemployment rate is also lower in future periods compared with the case of exogenous benefits.

V. B. An Example with a Succession of Bad Shocks

A different implication of the model from the previous literature concerns the effect of a succession of bad (or adverse) shocks. In a standard hysteresis model, unemployment should increase monotonically with the occurrence of negative shocks. This does not occur in the present model. Two cases are worth analyzing. In the first case, insurance effects of the shocks continue to dominate benefit setting, whereas in the second case tax effects begin to dominate.

The first case occurs when an economy gets stuck at a high level of unemployment because benefits get stuck at a high level after a shock. This is the standard case depicted by point D in Figure 1. Now imagine that a second, even bigger shock hits the economy. Social welfare falls below *S¢S¢* and, if insurance considerations prevail, the new maximum lies to the right of b^{e1}. Assume that it is optimal to adjust benefits up in the presence of this big shock. It is also true that the gain from adjusting benefits down will now be bigger than DS^{0}. Thus, a bigger shock may actually make it optimal to have an adjustment down of benefits.

A second case concerns bad shocks of such large magnitude that tax effects, rather than insurance effects, begin to dominate the government's benefit setting problem. First,

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14 In such a case the calculations we do below should incorporate the costs of inflation.

assume that a bad shock has driven up optimal benefits and, because of institutional adjustment costs, it is optimal not to reduce them once the shock disappears. Now assume that a further bad shock hits the economy that leaves unemployed a large proportion of the labor force. In the traditional hysteresis models there are no self-correcting mechanisms (see Hall (1986)). In our model, there always comes a point where the shock is sufficiently large that benefits must be directly reduced by the government due to its budget constraint. As before, let $b^0=argmax_b\, S(b,0)$ and $b^{\varepsilon 1}=argmax_b\, S(b,\varepsilon 1)$ where $\varepsilon 1>0$.

Proposition 6: As the unemployment rate, $u \rightarrow 1$, the benefit level that maximizes current period social welfare decreases in the presence of an adverse shock: $b^{\varepsilon 1}< b^0$.

Proof: See Appendix I. #

Appendix II contains simulations of the model. It shows the impact of different shocks on the optimal level of benefits, including when there exist fixed adjustment costs, for particular parameter values.

V. C. The Natural Rate of Unemployment

Work on the natural rate of unemployment defines it independently of aggregate demand conditions and the current rate of unemployment (Friedman (1968), Phelps (1968, 1994)).[15] Previous work on hysteresis has pointed out that this distinction may be overstated. Even if one rejects the behavioral assumptions on which those models stand, the difficulty in defining the natural rate remains once institutions are endogenized. Only if institutions (in our case benefits) are set exogenously can we define a natural rate of unemployment, u^n, independently of the temporary, random shocks affecting the economy: $u_t=\alpha b_t+\varepsilon_t \Rightarrow u^n=E(u_t)=\alpha b_t$ where $E(\varepsilon_t)=0$. On the other hand, if benefits are set optimally, then the "natural rate" will in general depend on the history of shocks to unemployment, via the

[15] Friedman (1968) defines the natural rate as *"the level which would be ground out by the Walrasian system of general equilibrium equations, provided that there is embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the cost of mobility, and so on."*

effect of these shocks on the level of benefits.: $u^n=E(u_t)=\alpha b(\varepsilon_t,\varepsilon_{t-1},\varepsilon_{t-2},\ldots)$.

VI. Conclusions

A number of economists have blamed European unemployment on labor market institutions. Since institutions are primitives in these models, a lot of the dynamics have been left unexplained. Consider for example, the time path of unemployment benefits. Figures 4 and 5 shows them increasing sharply in the US and Spain in the years immediately after 1973 and 1979. A similar pattern is present in the data for other OECD countries. If we believe institutions are exogenous, we must also believe that these countries were incredibly unlucky. Just when they got hit by an oil shock, politicians decided to increase benefits, worsening their unemployment problems. Only the US turned out to be lucky in the mid-1980's when benefits returned to their pre-shock levels. A less ad-hoc story involves developing a theory where institutions are rational. In such a theory, unemployment benefits can certainly increase the unemployment rate, but it should also allow us to understand what drives movements in benefits. This is the objective of our paper.

We present a model where the government sets unemployment benefits to maximize social welfare in response to an unemployment shock, subject to a budget constraint and the possibility that unemployment benefits may introduce incentive problems that increase the unemployment rate. The following results can be established:

1. In the absence of incentive effects (whereby higher benefits increase the unemployment rate) there should be full insurance. Unemployment benefits, on the other hand, should be set lowest (highest) when the adverse incentive effects of benefits are largest (smallest).
2. In response to a shock that increases unemployment, benefits should be increased in those economies where the adverse incentive effects are most severe. The intuition for this result stems from the fact that benefits are set optimally at all times, including the moment just before the shock occurs. Thus, large incentive effects imply a low initial level of benefits and large welfare gains derived from better insurance when there is an unemployment shock.
3. In the presence of an adjustment cost of changing the level of benefits there may exist hysteresis in benefit setting and unemployment. In other words, the level of

benefits (and unemployment) may rise in the presence of an adverse shock and remain higher than the initial value once the shock has disappeared. The reason for the asymmetry is that a shock increases the degree of *concavity* of the objective function (social welfare). This occurs because the shock incorporates into the objective function a group of people who are on a more concave part of their utility function. This suggests that the key assumption driving hysteresis is that the individual utility function has positive third derivative (people do not become more risk averse as they become richer).

4. As in previous models of hysteresis, temporary good shocks may now have permanent effects on unemployment. The reason is that lower unemployment may make lower benefits optimal. Contrary to previous models, we do not require any behavioural asymmetries between "insiders" and "outsiders" or between the short-term unemployed and the long-term unemployed.

Appendix I

Proof of Proposition 1

Let $SS(b,e)=\partial^2 S(b,e)/\partial b^2<0$.

a. Consider the outcome when e changes from 0 to $e1$ to 0. Define $f(z)=S(b^0+z,0)$ and $g(z)=S(b^{e1}-z,e1)\ \forall z \in (0,b^{e1}-b^0)$. Denote the first derivatives of these functions by $f'(z)$ and $g'(z)$, respectively. Then $[g'(z)-g'(0)]/[f'(z)-f'(0)]=g''(r)/f''(r)$ for some $r \in (0,z)$ by the Cauchy Mean Value Theorem (CMVT). But $g'(0)=f'(0)=0$ and $g''(r)<f''(r)<0$ because $SS(b^{e1}-z,e1)<SS(b^0+z,0)<0$. Hence $g'(z)/f'(z)>1$ $\forall z$. Using the CMVT again, $DS^{e1}/DS^0=[g(b^{e1}-b^0)-g(0)]/[f(b^{e1}-b^0)-f(0)]=g'(s)/f'(s)$ for some $s \in (0,b^{e1}-b^0)$. Hence $DS^{e1}/DS^0>1$ since $g'(s)/f'(s)>1$. If $DS^0<m<DS^{e1}$ then b changes from b^0 to b^{e1} (the gain equals $DS^{e1}-m>0$) but not back again (the loss would equal $DS^0-m<0$). Consequently there exists hysteresis.

b. Consider the outcome when e changes from $e1$ to 0 to $e1$ (i.e. the shock is of size $-e1$). Since $0>SS(b^{e1}-z,e1)>SS(b^0+z,0)$ then $DS^{e1}/DS^0<1$, by similar logic as in part (a). If $DS^{e1}<m<DS^0$ then b changes from b^0 to b^{e1} (the gain equals $DS^0-m>0$) but not back again (the loss would equal $DS^{e1}-m<0$). Consequently there again exists hysteresis. #

Proof of Proposition 3

Substituting in $SW(b,u,T,e)$, for constraints (8) and (9) yields $S(b,e)$. The effect of a shock on the marginal gain from increasing benefits is:

$$\frac{\partial^2 S}{\partial e \partial b}=U'(b)-U'(w)-\frac{\partial w^g}{\partial e}U'(w)[a-r\ (u+\frac{ab}{1-u})(1-\frac{b}{(\partial w^g/\partial e)(1-u)^2})] \qquad \text{(A1)}$$

where $\partial w^g/\partial e<0$ and $r=-U''(w)/U'(w)$ is the coefficient of absolute risk aversion.

a. As $a \to 0$, the FOC (10) implies that $U'(w) \to U'(b)$ and from (A1):

$$\frac{\partial^2 S}{\partial e \partial b}\to U'(w)\ r\ (u+\frac{ab}{1-u})(\frac{\partial w^g}{\partial e}-\frac{b}{(1-u)^2}) \qquad \text{(A2)}$$

which is negative. Hence using the implicit function theorem, benefits should be cut following the occurrence of an adverse shock when incentive effects are small.

b. If incentive effects are large so b is small then:

$$\frac{\partial^2 S}{\partial e \partial b}\to\ U'(b)-U'(w)-\frac{\partial w^g}{\partial e}U'(w)[a-r(u+\frac{ab}{1-u})] \qquad \text{(A3)}$$

which is positive provided that the utility function is strictly concave and the coefficient of absolute risk aversion, r, has an upper bound. Hence using the implicit function theorem, benefits should be increased following the occurrence of an adverse shock when incentive effects are large. #

Proof of Proposition 4

The second derivative of the social welfare function is:

$$\frac{\partial^2 S}{\partial b^2} = u\,U''(b) + 2\alpha[U'(b) - \frac{\partial w}{\partial b}U'(w)] + (1-u)[(\frac{\partial w}{\partial b})^2 U''(w) + \frac{\partial^2 w}{\partial b^2}U'(w)] \qquad \text{(A4)}$$

The effect of a shock on the concavity of the welfare function for a given value of b is:

$$\frac{\partial}{\partial \varepsilon}(\frac{\partial^2 S}{\partial b^2}) = U''(b) + \Phi U''(w) - \psi U'''(w)\frac{(u + \frac{\alpha b}{1-u})^2}{1-u} \qquad \text{(A5)}$$

where $\Phi=\alpha b[4+3\alpha b/(1-u)]/(1-u)^3+u(2-u)/(1-u)^2-2\alpha(\partial w^g/\partial \varepsilon)$ and $\psi=-\partial w^g/\partial \varepsilon+b/(1-u)^2$. Since both Φ and ψ are positive, $U''(b)<0$ and $U''(w)<0$, a sufficient condition for (A5) to be negative is that $U'''(w)\geq 0$ (which is the case for all quadratic, Constant Absolute and Constant Relative Risk Aversion functions). Hence $\partial^2 S/\partial b^2$ is a monotonically decreasing function of ε so $\partial^2 S(b,\varepsilon 1)/\partial b^2<\partial^2 S(b,0)/\partial b^2 \quad \forall b, \ \forall \varepsilon 1>0.$ #

Proof of Proposition 5

It is simple, but not necessary, to let $\theta \to \infty$ so that only current period welfare is valued by the government. In the presence of the shock, welfare changes by $\Delta S^{\varepsilon 1}-\mu$ if benefits are adjusted from b^0 to $b^{\varepsilon 1}$. After the shock has gone, welfare changes by $\Delta S^0-\mu$ if benefits are adjusted back from $b^{\varepsilon 1}$ to b^0. Proposition 4 states that the concavity of the welfare function increases (i.e. becomes more negative) in the presence of shocks, $\varepsilon 1>0$: $\frac{\partial^2 S(b,\varepsilon 1)}{\partial b^2} < \frac{\partial^2 S(b,0)}{\partial b^2} < 0 \ \forall b.$ Provided $|b^{\varepsilon 1}-b^0|$ is small, then the condition in Proposition 1(a) will be satisfied and hence there exists hysteresis whenever adjustment costs lie in the range, $\Delta S^0<\mu<\Delta S^{\varepsilon 1}$. An estimate of the difference, $\Delta S^{\varepsilon 1}-\Delta S^0$, is: $-\frac{1}{2}[\frac{\partial^2 S(b,\varepsilon 1)}{\partial b^2} - \frac{\partial^2 S(b,0)}{\partial b^2}](b^{\varepsilon 1}-b^0)^2 > 0$ (using 2nd order Taylor approximations). #

Proof of Proposition 6

As $u \to 1$ in equation (A1), $\partial^2 S/\partial \varepsilon \partial b$ becomes negative (for given α). Hence using the implicit function theorem, $db/d\varepsilon<0$, and so $b^{\varepsilon 1}< b^0$. #

Appendix II

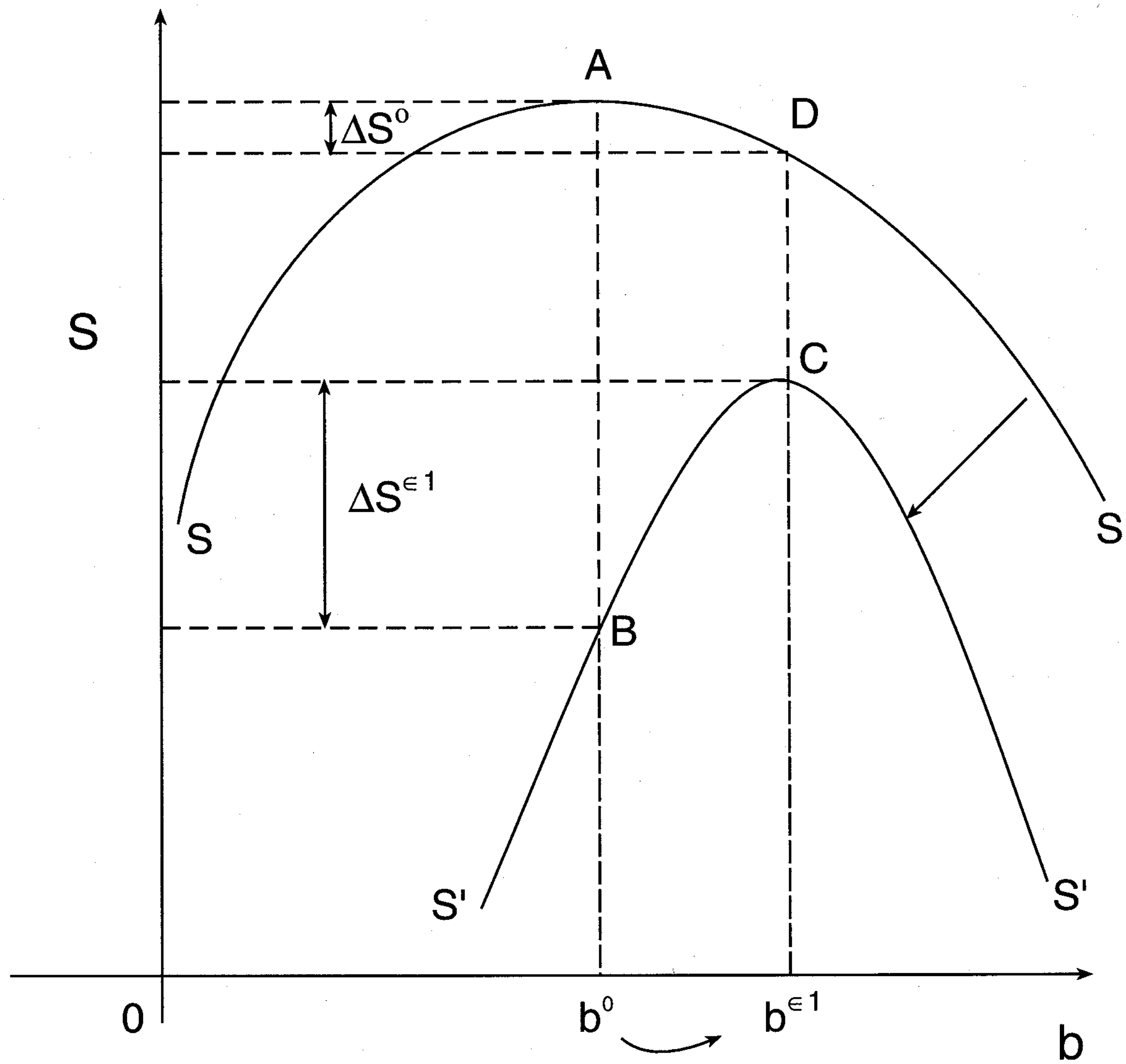

Figure 1: The objective function, *S(b, e)*, versus the choice variable, *b*, before a shock (*SS*) and during an adverse shock (*S¢S¢*).

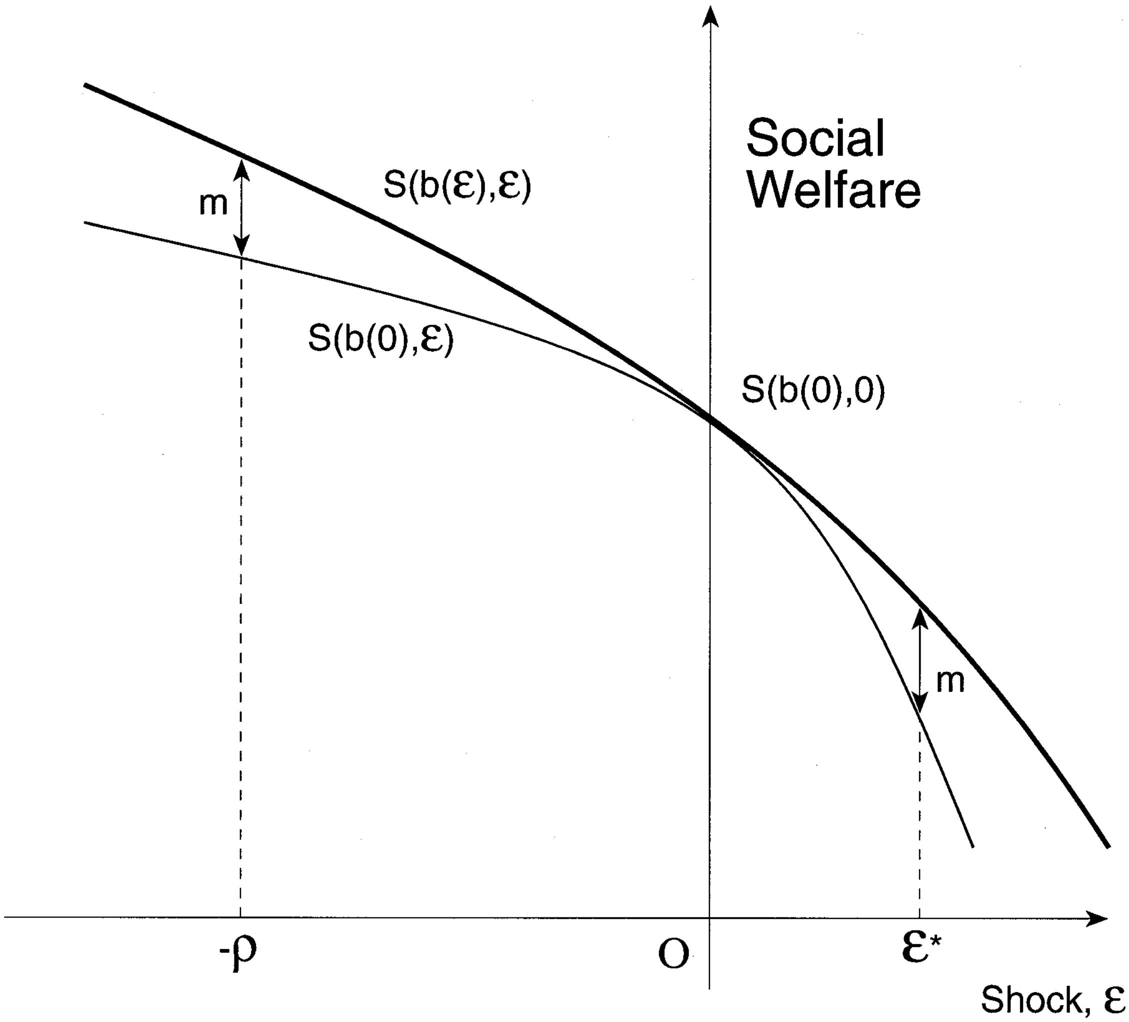

Figure 2 Social Welfare versus Unemployment Shocks. *S(b(e),e)* is the envelope over which benefits are changed optimally depending on the size of the shock. When the adjustment cost is *m*, the corresponding region of inaction is *(-r,e*)*.

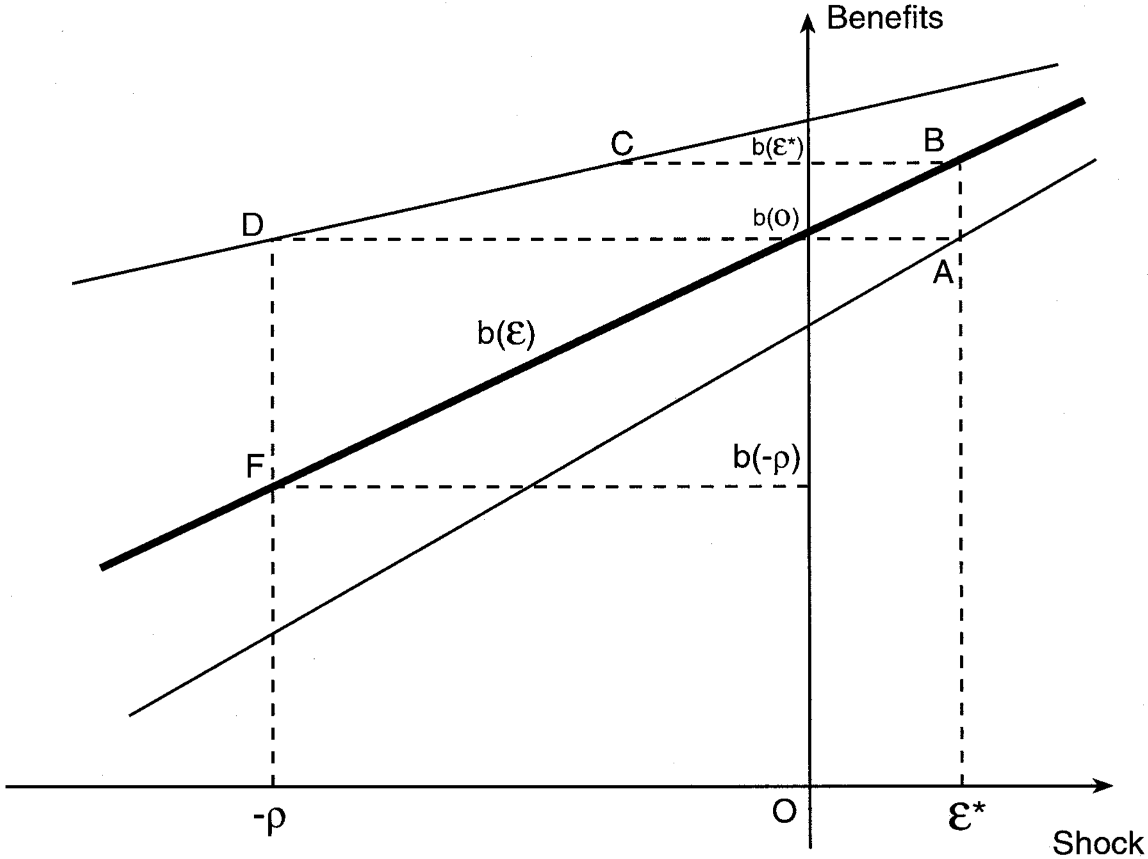

Figure 3: The Optimal *(S,s)* Unemployment Benefit Setting Rule. Segment DA denotes the (asymmetric) region of inaction.

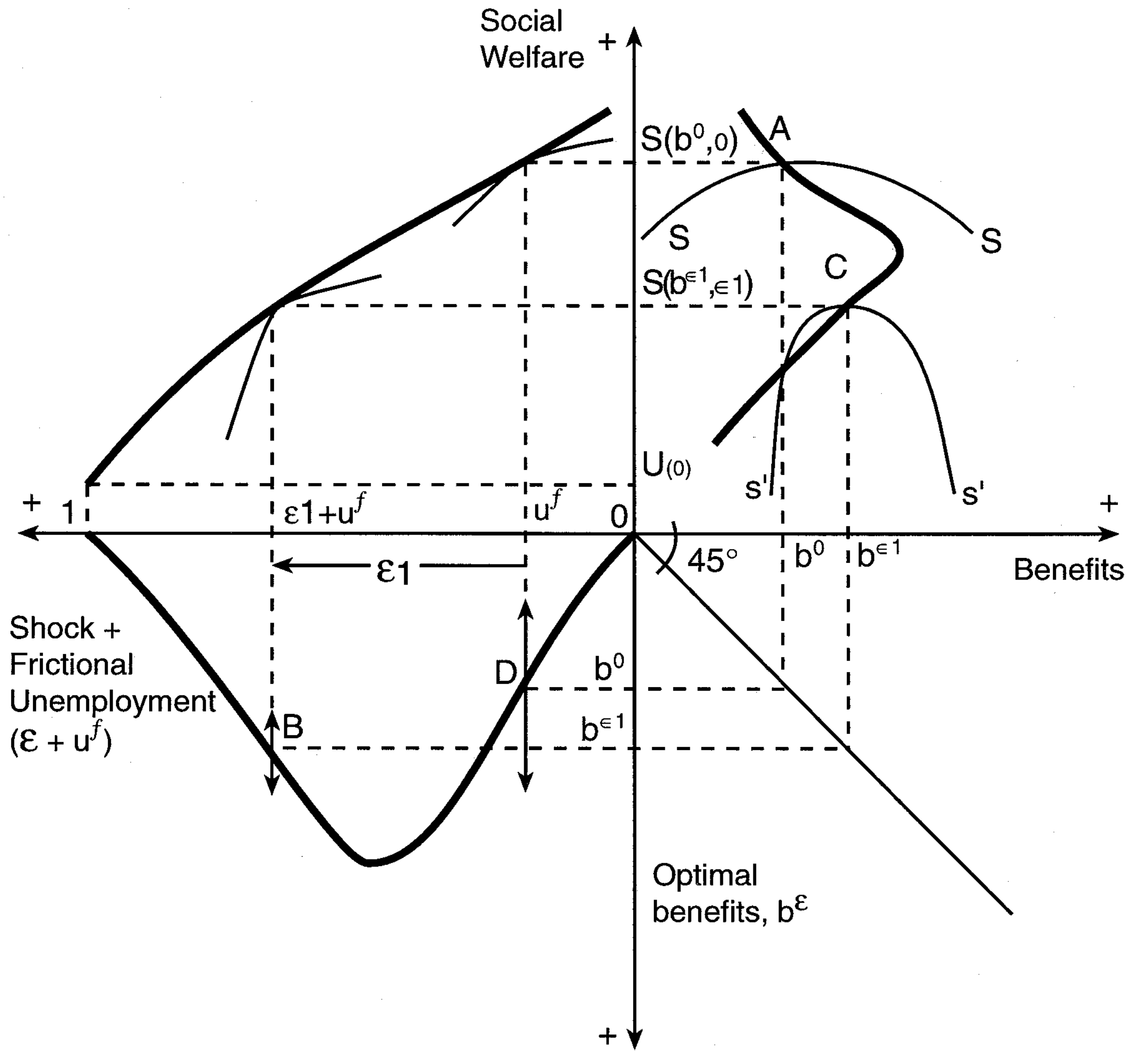

Figure 4: A Four Quadrant Summary of the Model. The thick lines show how social welfare varies with optimal benefits (as the shock varies) and how both welfare and optimal benefits vary directly with the size of the shock.

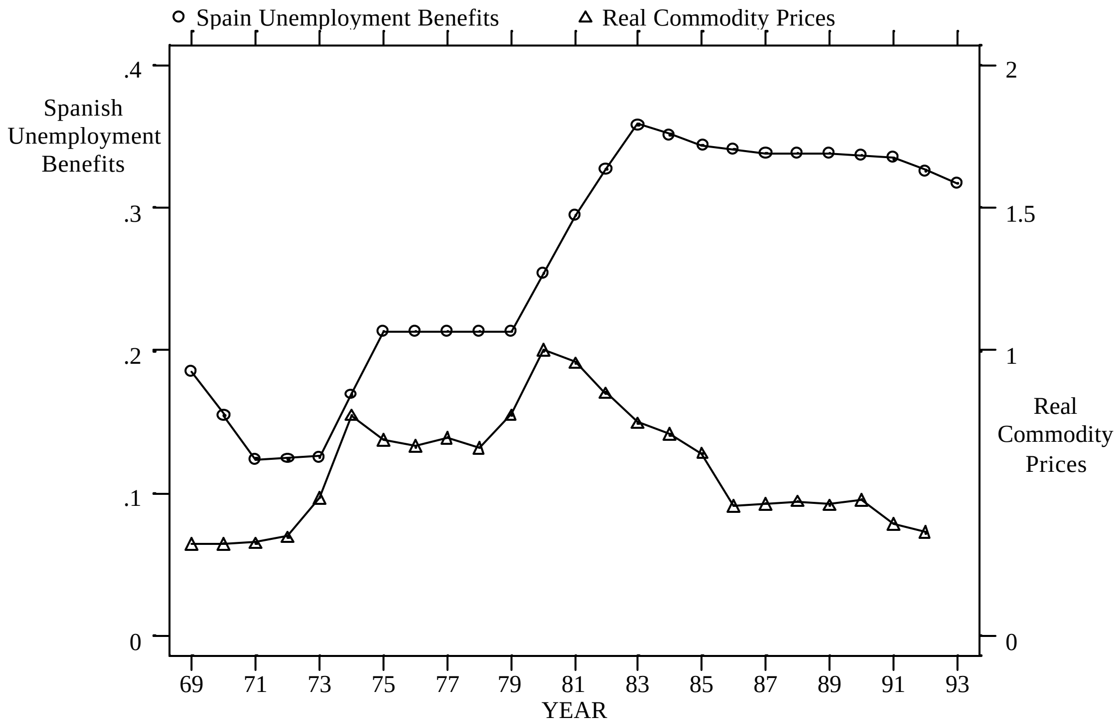

Figure 5: Spain's Unemployment Benefits and Real Commodity Prices from 1969 to 1993.

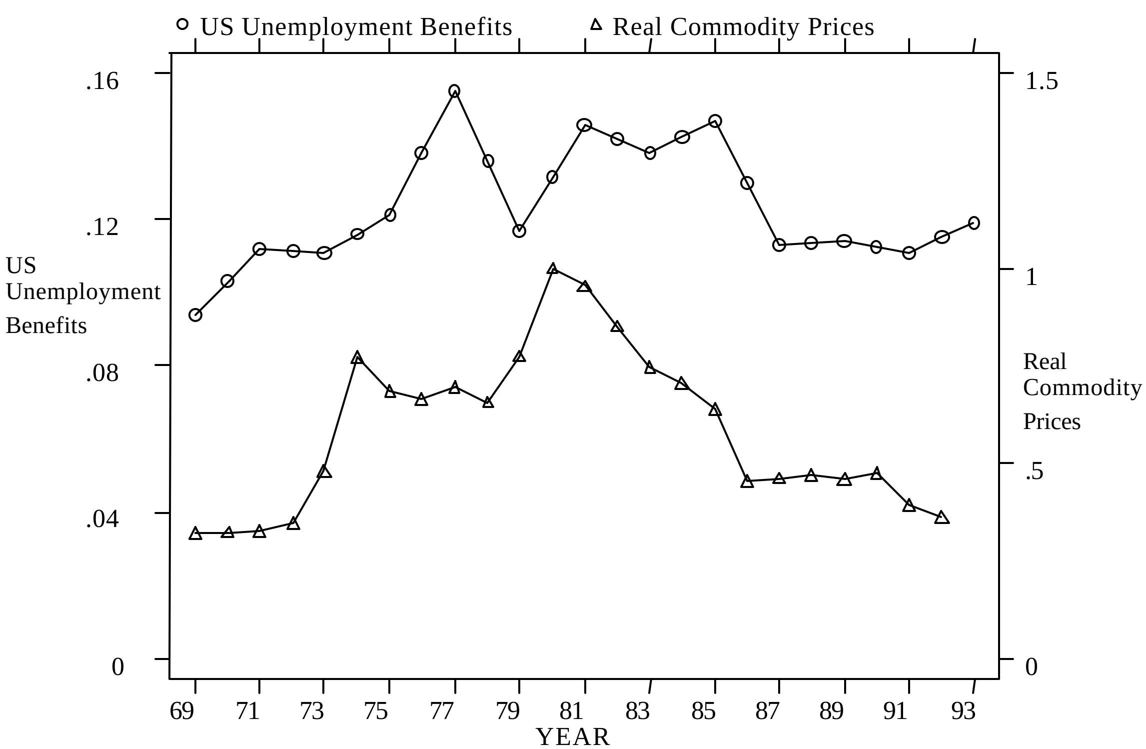

Figure 6: The United State's Unemployment Benefits and Real Commodity Prices from 1969 to 1993.

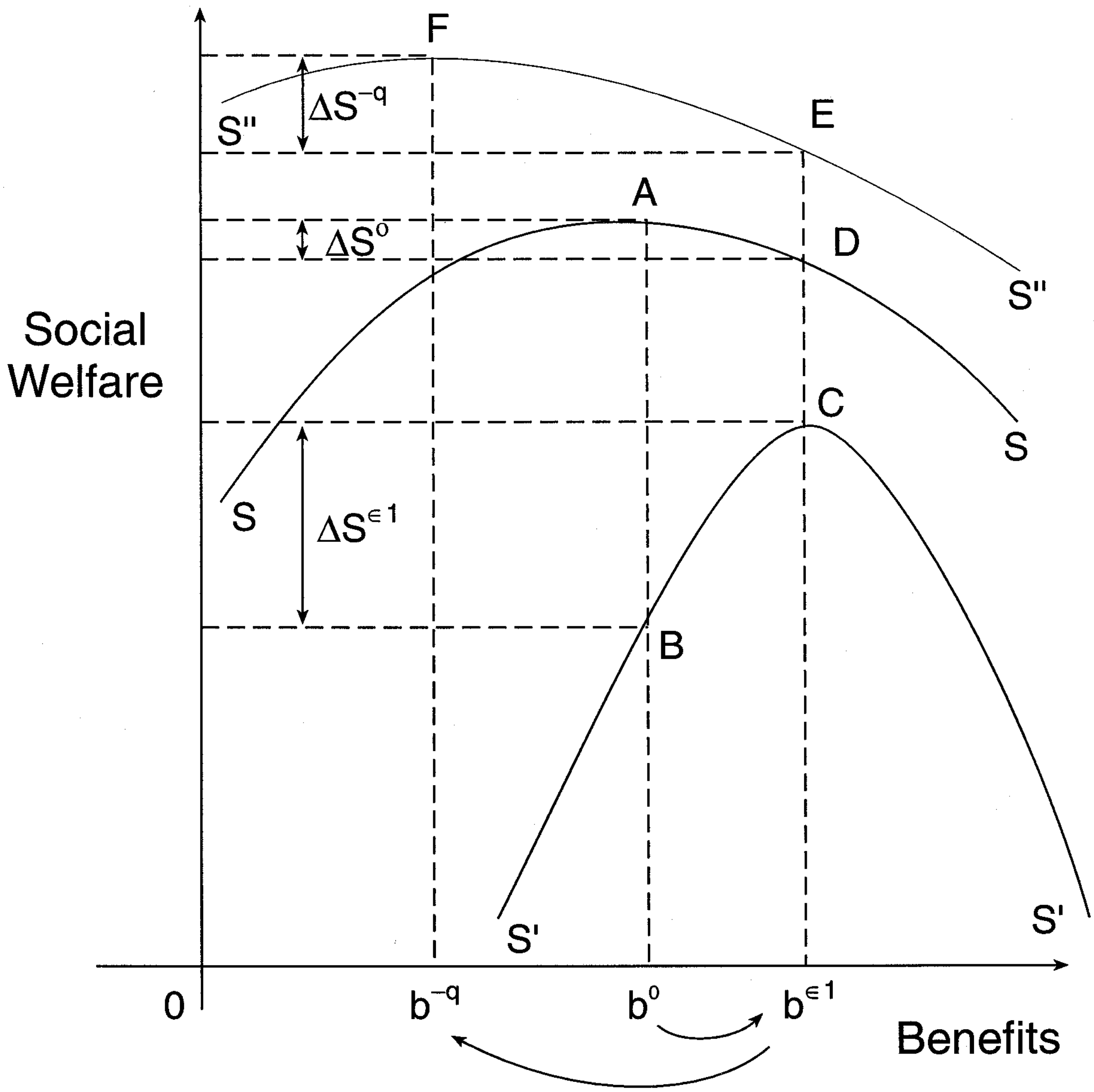

Figure 7: Social Welfare versus Unemployment Benefits before a shock (*SS*), during an adverse shock to unemployment (*S¢S¢*) and during a positive shock that reduces unemployment (*S¢¢S¢¢*).

Appendix III

A Numerical Simulation

Let utility be logarithmic, w^g=1, u^f=0.04, a=0.2 and the government's rate of time preference be high so that it only values current period welfare. Assume there are no adjustment costs. The optimal level of benefits, b^0, equals 0.13, and the unemployment rate equals 0.07. Social welfare, $S(b^0,0)$=-0.14. Consider the effect of an adverse unemployment shock, $e1$, of size 0.04. For simplicity, let the gross wage be unaffected by the shock. If benefits remain at their pre-shock level then welfare drops to $S(b^0,e1)$=-0.23. However if they are increased to b^{e1}=0.32, which maximizes $S(b,e1)$, then welfare would be $S(b^{e1},e1)$=-0.21. In other words, welfare can be increased by $DS^{e1}=S(b^{e1},e1)-S(b^0,e1)$=0.02 by increasing benefits. After the shock has gone, the gain from reducing benefits from b^{e1} back to their initial value, b^0, equals $DS^0=S(b^0,0)-S(b^{e1},0)$= 0.01. Hence the welfare gain from increasing benefits in the presence of the shock is twice the size of the gain from reducing them after the shock has gone.

Optimal Benefits with Adjustment Costs

Now assume that adjustment costs satisfy 0.01<m<0.02. In such a case, it is worthwhile for the government to raise benefits in the period that the temporary shock occurs but not to reduce them once e=0. If m=0.015 then in the period of the shock, social welfare can be increased by 0.005 ($DS^{e1}-m$=0.02-0.015) by raising benefits from b^0=0.13 to b^{e1}=0.32. In the period that the shock disappears, benefits should not be cut since this policy would result in a welfare loss equal to –0.005 (DS^0-m=0.01-0.015). The unemployment rate also does not return to its initial value, due to the higher level of benefits. After the shock has disappeared, the unemployment rate equals 0.10, which is 0.03 higher than its initial (pre-shock) value.

Good Shocks

Start from the position where benefits have been optimally increased to b^{e1}=0.32 following the adverse shock of size, $e1$=0.04, and have become stuck at this level due to the adjustment cost, m=0.015. Social welfare, $S(b^{e1},0)$=-0.15. Now consider the effect of a good shock that temporarily reduces unemployment by $-q$=-0.02. Social welfare rises by 0.02 as a direct consequence (=$S(b^{e1},-q)-S(b^{e1},0)$=-0.13-(-0.15)). By cutting benefits to b^{-q}=0.07, welfare can be further increased by 0.01 (=$DS^{-q}-m=S(b^{-q},-q)-S(b^{e1},-q)-m$=-0.105-(-0.13)-0.015). In the absence of further shocks, benefits will permanently remain at this lower level. Unemployment also remains low, equal to 0.05, compared to its pre-shock value of 0.10.

A Succession of Bad Shocks

Start from the position in which benefits have been optimally increased from b^0=0.13 to b^{e1}=0.32 following the first adverse shock of size, $e1$=0.04, and have become stuck at this level due to the adjustment cost, m=0.015. Social welfare, $S(b^{e1},0)$=-0.15. Now consider the effect of another bad, but larger shock to unemployment of size, $e2$=0.16. If benefits are not changed then welfare drops to $S(b^{e1},e2)$=-0.45. However if benefits are increased further to b^{e2}=0.46, which maximizes $S(b,e2)$, then welfare can be increased above this level by 0.005 ($S(b^{e2},e2)-S(b^{e1},e2)-m$=-0.43-(-0.45)-0.015). After the second shock has gone, it is worthwhile to cut benefits back to their initial value, b^0=0.13, since the gain from doing so, even after paying the adjustment cost, is positive. It equals 0.015 (=$S(b^0,0)-S(b^{e2},0)-m$=-0.14-(-0.17)-0.015=0.015). Unemployment also returns to its initial value, equal to 0.07.

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