MEASURING CONSUMER AND COMPETITIVE IMPACT WITH ELASTICITY DECOMPOSITIONS

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In this article, I discuss three methods of decomposing the elasticity of own-good demand. One of the methods, the decision-based decomposition (Gupta, 1988), is useful in determining the influence of changes in consumers' decisions on the growth in own-good demand. The other two methods, the unit-based decomposition (van Heerde et al., 2003) and the share-based decomposition (Berndt et al., 1997), are useful in determining whether the growth in own-good demand has been stolen from competing goods.

The objective of this article is to provide a clear and accurate method that attributes the growth in own-good demand to changes in: (1) consumers' decisions, (2) competitive demand, and (3) competitive market share. I will accomplish this by settling some confusion about what the decision- and share-based decompositions mean, by discussing how each of the decompositions relate to the others, and by discussing the research questions that each of the decompositions can answer.

I. Introduction

Three methods of decomposing the elasticity of demand have been used to study whether marketing actions expand the market or steal business from rival firms. These decompositions, when applied to the same problem, produce seemingly contradictory results. One method, for example, may suggest that all of the demand created by an incremental advertising investment would be generated by market expansion while another suggests the same increase would be stolen from rival firms. I will explain why these apparently contradictory results actually are complementary and provide a more comprehensive understanding of the investment's impact.

Consider the following example. Suppose two firms are competing in a market. Firm A is considering whether to increase its advertising investments by a small amount. The unit sales and market shares that would be earned by the firms at the two investment levels are summarized in Table One. Firm A would like to know whether the growth in its demand comes at the expense of Firm B.

< *Insert Table One* >

Several methods have been developed to answer this question. The crucial difference among them lies in how the researchers measure stolen business. Some authors measure stolen business by the decrease in demand for competing goods (van Heerde et al., 2003; van Heerde et al., 2004). I will refer to these methods as *unit-based decompositions*. In our example, these authors would claim that none of the growth in Firm A's demand would come at the expense of Firm B because Firm B's demand would not be affected by the advertising investment. This is a reasonable point of view to take.

Other authors measure stolen business by the decrease in market share of competing goods (Berndt et al., 1995; Berndt et al., 1997; Rosenthal et al., 2003). I will refer to these methods as *share-based decompositions*. In our example, these authors would claim that some of the growth in Firm A's demand would come at the expense of Firm B because Firm B's market share would drop by 1.3 percentage points due to the advertising investment. This too is a reasonable point of view to take.

Despite appearing to offer very similar measures of stolen business, the unit- and share-based methods can produce strikingly different results. In our example, the unit-based measure suggests that none of the growth in Firm A's demand would come at the expense of Firm B, but the share-based measure suggests that two-thirds of it would. Firm B would need to earn 1,020 units in the expanded market in order to maintain its original market share. Since it earns only 1,000 units, the share-based measure classifies 20 units of the 30 unit increase as being stolen from it.

Not only do these decompositions give very different impressions of what is happening in the marketplace, the share-based method has not been fully understood. Berndt et al. (1997) write:

> We distinguish between two types of marketing: (1) that which concentrates on bringing new customers into the market ("[market]-expanding" advertising), and (2) that which concentrates on competing for market shares from these consumers ("rivalrous" advertising).

This interpretation is misleading. Share-based decompositions classify only a portion of the market expansion as being primary (or non-rivalrous) demand. In our example, the share-based decomposition classifies only 10 units of the 30 unit increase in market demand as primary. By contrast, the unit-based measure defines primary demand to be equivalent to the market expansion, the full 30 unit increase.

A third set of authors study a marketing action's impact from an entirely different perspective. They measure the influence of changes in the consumers' decisions on the growth in own-good demand (Gupta, 1988; Chiang, 1991; Chintagunta, 1993; Bucklin et al., 1999; and Bell et al., 1999). I will refer to these as *decision-based decompositions*.[2] These decompositions, contrary to some suggestions otherwise, are insufficient to determine a marketing investment's competitive impact because they measure changes only in own-good demand, not in competitive demand. Nevertheless, I will show how to extend a decision-based analysis to competing goods by decomposing the elasticity of cross-good demand.

The objective of this article is to provide a clear and accurate method that attributes the growth in own-good demand to changes in: (1) consumers' decisions, (2) competitive demand, and (3) competitive market share. I will accomplish this by settling some confusion about what the decision- and share-based decompositions mean, by discussing how each of the decompositions relates to the others, and by discussing the research questions that each of the decompositions can answer. From the unit-based decomposition, a brand manager can learn whether the growth in own-good demand is due to stolen units. From the share-based decomposition, a manager can learn whether it is due to stolen market share. From the decision-based decomposition, a manager can learn which changes in consumer behavior lead to the growth in demand. Used together, these methods provide a comprehensive understand of a marketing investment's impact.

The remainder of the paper is organized as follows. In section two, I derive the relationship between the unit- and share-based decompositions. Contrasting the two

[2] I will show that any of the decompositions, even the unit-based one, can be derived from the elasticity of demand. Therefore, I will eschew using the term elasticity decomposition to distinguish the decision-based decomposition from the others.

decompositions clarifies the perspective that each method offers and coerces a more precise interpretation of the share-based method. I illustrate the difference between the methods using an example based on the empirical results of Berndt et al. (1997). In section three, I decompose the elasticity of cross-good demand to isolate the impact of each consumer decision on competitive demand. This analysis resolves the discrepancies in the coffee example of van Heerde et al. (2003) and clarifies the meaning of decision-based decompositions. In section four, I derive the relationships between the decision-based and the unit- and share-based decompositions using the previous cross-good analysis. This allows me to construct matrices that fully account for how a marketing action affects both consumers' decisions and the demand for and market-share of competing goods. I illustrate the unified decompositions by returning to the coffee example and discuss a paradox that can arise when the own-good market share is low. I conclude in section five. All of the decompositions that will be discussed measure an investment's contemporaneous effects.

2. Decompositions that Measure Competitive Impact

Let's begin by comparing the unit- and share-based approaches to studying a marketing action's competitive impact. Both methods attribute the growth in own-good demand to rivalrous and non-rivalrous sources. The unit-based method measures stolen business by the decrease in demand for competing goods whereas the share-based method measures stolen business by the decrease in their market share. Neither method requires a model of the consumers' decision-making process in order to make this judgment. I will show that both decompositions accurately depict how a marketing action would affect competing goods and will explain how to interpret differences in their results.

Let's begin with some notation. Let q_j represent the demand for good *j*,

$Q_{-j} = \sum_{\substack{k=1 \\ k \neq j}}^{J} q_k$ represent the demand for competing goods (competitive demand), and

$Q_{all} = \sum_{k=1}^{J} q_k$ represent the demand for all goods in the market (market demand).

Similarly, let s_j represent the market share of good *j* and $S_{-j} = \sum_{\substack{k=1 \\ k \neq j}}^{J} s_k$ represent the share of competing goods. Let m_j be a marketing instrument for good *j*. The elasticities of demand are $\eta_{q_j, m_j} = \frac{\partial q_j}{\partial m_j} \cdot \frac{m_j}{q_j}$, $\eta_{Q_{-j}, m_j} = \frac{\partial Q_{-j}}{\partial m_j} \cdot \frac{m_j}{Q_{-j}}$, and $\eta_{Q_{all}, m_j} = \frac{\partial Q_{all}}{\partial m_j} \cdot \frac{m_j}{Q_{all}}$ and of share are $\eta_{s_j, m_j} = \frac{\partial s_j}{\partial m_j} \cdot \frac{m_j}{s_j}$ and $\eta_{S_{-j}, m_j} = \frac{\partial S_{-j}}{\partial m_j} \cdot \frac{m_j}{S_{-j}}$.

2.1 Unit-Based Decompositions

Unit-based decompositions measure stolen business by the decrease in demand for competing goods. These decompositions are derived from the identity

$$q_j = Q_{all} - Q_{-j}. \tag{1}$$

Demand for the target good is expressed as the difference between demand for all goods in the market and demand for competing goods.

The impact of an incremental marketing investment is quantified by taking derivatives, such that

$$\frac{\partial q_j}{\partial m_j} = \frac{\partial Q_{all}}{\partial m_j} - \frac{\partial Q_{-j}}{\partial m_j}. \tag{2}$$

This equation attributes the growth in own-good demand to two sources. The non-rivalrous source is measured by the increase in market demand, $\partial Q_{all} / \partial m_j$. The rivalrous source is measured by the decrease in demand for competing goods, $\partial Q_{-j} / \partial m_j$.

Figure One provides a geometric depiction of the unit-based decomposition. As the result of an incremental marketing investment, the market expands from A to A' and the demand for competing goods contracts from B to B'. Line segment $\overline{AB}$ represents the demand for good *j* prior to the incremental marketing investment and line segment $\overline{A'B'}$ represents demand afterwards. Demand for good *j* grows by $\overline{A'C}$ + $\overline{DB'}$ units. Of the growth, $\overline{A'C}$ units are generated by market expansion and $\overline{DB'}$ units are stolen from competing goods. Demand for competing goods decreases by $\overline{DB'}$ units. For small $\delta = m'_j - m_j$, the quantities represented by the line segments are $\overline{A'C} = \frac{\partial Q_{all}}{\partial m_j} \cdot \delta$ and $\overline{DB'} = -\frac{\partial Q_{-j}}{\partial m_j} \cdot \delta$.

< Insert Figure One >

Unit-based decompositions can be transformed from derivatives into elasticities by multiplying all terms by m_j / q_j . This transformation results in

$$\eta_{q_j, m_j} = \left(Q_{all} / q_j \right) \cdot \eta_{Q_{all}, m_j} - \left(Q_{-j} / q_j \right) \cdot \eta_{Q_{-j}, m_j} . \tag{3}$$

The leading term Q_{all} / q_j simply scales the change in market demand from being measured relative to the level of market demand to being measured relative to the level of demand for good *j*. Similarly, the leading term Q_{-j} / q_j scales the change in demand for competing goods from being measured relative to the level of demand for competing goods to being measured relative to the level of demand for good *j*.

The proportions

$$\Psi_{\substack{\text{market}\\ \text{expansion, j}}} = \frac{\eta_{Q_{all},m_j} \cdot Q_{all}}{\eta_{q_j,m_j} \cdot q_j} \quad \text{and} \tag{4}$$

$$\Psi_{\substack{\text{stolen}\\ \text{units, j}}} = \frac{\eta_{Q_{-j},m_j} \cdot Q_{-j}}{\eta_{q_j,m_j} \cdot q_j} \tag{5}$$

provide unit-based measures of primary and secondary demand. The following interpretation applies: If own-good demand were to grow by 100 units following a marketing investment, $\Psi_{\substack{\text{market}\\ \text{expansion, j}}} *100$ of the units would be created by market expansion and $\Psi_{\substack{\text{stolen}\\ \text{units, j}}} *100$ of the units would be stolen from competing goods. Demand for competing goods would decrease by $\Psi_{\substack{\text{stolen}\\ \text{units, j}}} *100$ units.

The proportion of growth in own-good demand that is created by market expansion is not restricted to be less than one. A value greater than one, however, does not imply that more than 100% of the growth comes from market expansion. Rather, it implies the marketing investment creates $\left(\Psi_{\substack{\text{market}\\ \text{expansion, j}}} -1\right)$ units of demand for competing goods for every unit that it creates for the target good. For example, a value of $\Psi_{\substack{\text{market}\\ \text{expansion, j}}} =1.5$ implies that an advertising investment that creates 100 units of demand for the target good also creates 50 units of demand for competing goods.

2.2. Share-Based Decompositions

Share-based decompositions measure stolen business by the decrease in market share of competing goods. These decompositions are derived from the identity

$$q_j = Q_{all} \cdot s_j \,. \tag{6}$$

Demand for the target good is expressed as the product of market demand and the target good's market share.

The impact of an incremental marketing investment is quantified by applying the chain rule, such that $\frac{\partial q_j}{\partial m_j} = s_j \cdot \frac{\partial Q_{all}}{\partial m_j} + Q_{all} \cdot \frac{\partial s_j}{\partial m_j}$. This equation is better expressed as

$$\frac{\partial q_j}{\partial m_j} = s_j \cdot \frac{\partial Q_{all}}{\partial m_j} - \left[\frac{\partial Q_{-j}}{\partial m_j} - S_{-j} \cdot \frac{\partial Q_{all}}{\partial m_j} \right]. \tag{7}$$

Equation (7) attributes the growth in own-good demand to two sources. The non-rivalrous source is $s_j \cdot \left(\partial Q_{all} / \partial m_j \right)$, a portion of the market expansion. The rivalrous source is defined as the demand that competing goods would need to regain in order to maintain their market share in the expanded market. Competing goods lose market share in the expanded market for two reasons: they lose units to the target good, $\partial Q_{-j} / \partial m_j$, and they fail to capture part of the expanded market, $-S_{-j} \cdot \left(\partial Q_{all} / \partial m_j \right)$.

The conceptual hurdle is recognizing that the share-based measure of primary demand is not equivalent to the demand generated by market expansion. Under share-based decompositions, marketing investments must create demand for competing goods in proportion to their market share in order to be considered non-rivalrous. This implies it is possible for marketing investments to create some demand for competing goods (investment spillover occurs), yet some of the growth in own-good demand is still classified as being stolen from them. While the concept of stolen market share is immediately understood, its implication on primary demand is more subtle.

Figure Two provides a geometric depiction of the share-based decomposition. As the result of an incremental marketing investment, the market expands from A to A' and demand for competing goods contracts from B to B'. Line segment $\overline{AB}$ represents demand for good *j* prior to the incremental investment and line segment $\overline{A'B'}$ represents it afterwards. Demand for good *j* grows by $\overline{A'EC}$ + $\overline{DB'}$ units. For market shares to be preserved, the ratio of $\overline{A'E}$ to $\overline{A'C}$ is equivalent to the ratio of $\overline{AB}$ to $\overline{AF}$. Of the growth in demand for good *j*, $\overline{A'E}$ units are defined as non-rivalrous and $\overline{EC}+\overline{DB'}$ units are generated by stealing share from competing goods. Demand for competing goods decreases by $\overline{DB'}$ units. For small $\delta = m_j' - m_j$, the quantities represented by the line segments are $\overline{A'E} = s_j \cdot \frac{\partial Q_{all}}{\partial m_j} \cdot \delta$, $\overline{EC} = S_{-j} \cdot \frac{\partial Q_{all}}{\partial m_j} \cdot \delta$, and $\overline{DB'} = -\frac{\partial Q_{-j}}{\partial m_j} \cdot \delta$.

< Insert Figure Two >

Expressed in terms of elasticities, the share-based decomposition is

$$\eta_{q_j,m_j} = s_j \cdot \eta_{Q_{all},m_j} - \left[\left(Q_{-j} / q_j \right) \cdot \eta_{Q_{-j},m_j} - S_{-j} \cdot \left(Q_{all} / q_j \right) \cdot \eta_{Q_{all},m_j} \right]. \tag{8}$$

The proportions

$$\Theta_{\substack{\text{share-preserving}\\ \text{market expansion, j}}} = \frac{s_j \cdot \left(\eta_{Q_{all},m_j} \cdot Q_{all} \right)}{\eta_{q_j,m_j} \cdot q_j} \qquad \text{and} \tag{9}$$

$$\Theta_{\substack{\text{stolen}\\ \text{share, j}}} = \frac{-\left(\eta_{Q_{-j},m_j} \cdot Q_{-j} \right) + S_{-j} \cdot \left(\eta_{Q_{all},m_j} \cdot Q_{all} \right)}{\eta_{q_j,m_j} \cdot q_j} \tag{10}$$

provide share-based measures of primary and secondary demand. These measures are related to those of the unit-based decomposition through the expressions

$$\Theta_{\substack{\text{share-preserving}\\ \text{market expansion, j}}} = s_j \cdot \Psi_{\substack{\text{market}\\ \text{expansion, j}}} \quad \text{and} \tag{11}$$

$$\Theta_{\substack{\text{stolen}\\ \text{share, j}}} = \Psi_{\substack{\text{stolen}\\ \text{units, j}}} + S_{-j} \cdot \Psi_{\substack{\text{market}\\ \text{expansion, j}}}. \tag{12}$$

This relationship should be expected. Both decompositions attribute the growth in own-good demand to changes in demand for competing goods. The share-based decomposition, however, attributes the demand competing goods would earn in an expanded market if they kept their market share, $S_{-j} \cdot \Psi_{\substack{\text{market} \\ \text{expansion, j}}}$, to the rivalrous source.

The unit- and share-based decompositions are simply different measures of competitive impact, and one can be recovered from the other without consideration of the consumers' decision-making process. All information needed to determine these decompositions is contained in the elasticities of own- and cross-good demand.

The following interpretation applies to share-based measures of primary and secondary demand: If own-good demand were to grow by 100 units following a marketing investment, $\Theta_{\substack{\text{share-preserving} \\ \text{market expansion, j}}} *100$ of these units would be created by share-preserving market expansion and $\Theta_{\substack{\text{stolen} \\ \text{share, j}}} *100$ of these units would reduce the market share of competing goods. Competing goods would need to take back $\Theta_{\substack{\text{stolen} \\ \text{share, j}}} *100$ units in the expanded market in order to maintain their market share and would need to take back $\Psi_{\substack{\text{stolen} \\ \text{units, j}}} *100$ units to maintain their demand.

2.3 Empirical Example – Berndt et al. (1997)

Berndt et al. (1995, 1997) study the growth and changing composition of the U.S. anti-ulcer drug market. Peptic ulcer disease occurs in 10-15 percent of the U.S. population and involves the inflammation of tissue in the digestive tract that is exacerbated by the presence of the body's naturally occurring gastric acid. SmithKline introduced Tagamet, a revolutionary treatment known as H_2-receptor antagonists, in August of 1977. Glaxo followed suit with Zantac in June of 1983, Merck with Pepcid in

October of 1986, and Lilly with Axid in April of 1988.

Berndt et al. (1995, 1997) estimate a system of two equations to describe consumer demand for these drugs. They specify a log-linear demand equation to describe the relationship between the market (industry) demand and the firms' marketing investments. They also specify a relative demand equation to describe the relationship between the firms' relative market shares and the relative investments made in support of their drugs. I will use Berndt et al.'s (1997) estimates from the two-product market that contains Tagamet and Zantac.[3] The elasticity of market demand is -0.268 for a change in the price of Tagamet and -0.804 for a change in the price of Zantac. The own-good elasticity of demand is -1.154 for Tagamet and is -1.690 for Zantac.

The share- and unit-based measures of primary and secondary demand are given in Table Two. The results of these methods provide very different impressions of whether price cuts steal business. Regardless of whether Tagamet or Zantac cuts its price, the unit-based measure suggests most of the growth in own-good demand comes from primary demand (92.9% for Tagamet and 63.4% for Zantac) whereas the share-based measure suggests most of the growth comes from secondary demand (76.8% for Tagamet and 52.4% for Zantac).

< *Insert Table Two* >

The decompositions, of course, are describing the competitive impact of the same price cut and should be interpreted as follows: A 1% decrease in Tagamet's price would yield a 1.154% increase in its demand. From the unit-based decomposition, we can say

[3] Parameter estimates for the market-level equation are found in column 2 of Table 7.1 on p. 301 and estimates for the market-share equation are found in column 4 of Table 7.2 on p. 307 of Berndt et al. (1997).

92.9% of the growth in Tagamet's demand would come from market expansion and 7.1% would be stolen from Zantac. This implies Zantac would have to take back 0.071 * 0.01154 * $q_{Tagamet}$ units from Tagamet in order to maintain its demand. From the share-based decomposition, we can say Zantac would need to take back 76.8% of the growth in demand for Tagamet, which amounts to 0.768 * 0.01154 * $q_{Tagamet}$ units, in order to maintain its market share in the expanded market. A similar analysis would apply to the growth in demand for Zantac if it were to cut its price.

The unit- and share-based methods provide complementary measures of the marketing action's competitive impact. The unit-based measure implies that only a small portion of the growth in Tagamet's demand would erode Zantac's demand, but the share-based measure implies that most of the same growth would erode Zantac's market share. One measure may be favored over the other depending on the brand manager's beliefs about what would trigger a competitive response from Zantac, lost demand or lost market share. Used in tandem, however, the measures provide the manager with a more complete understanding of whether the growth in Tagamet's demand has been stolen from Zantac.

3. Decompositions that Measure Consumer Impact

Decision-based decompositions measure the relative influence of changes in consumers' decisions on the change in demand for goods. Gupta (1988) shows how to measure the influence of these decisions on own-good demand. I will extend his analysis to measure their influence on competitive and market demand.

3.1 Decision-Based Decompositions

Decision-based decompositions require a model of the consumers' decision-making process, so let's begin by specifying a traditional model. Assume own-good demand is the product of three decisions: whether to purchase (incidence), which good to purchase if a purchase is made (conditional choice), and how much to purchase if a particular good is chosen (conditional quantity). The expected demand for good *j* is

$$q_j = N \cdot u \cdot v_j \cdot w_j \quad \forall j , \tag{13}$$

where *N* is the number of shopping occasions, u is the probability of buying in the category, v_j is the probability of choosing good *j* conditional on buying in the category, and w_j is the expected units purchased conditional on good *j* being chosen.

As has been shown (Gupta, 1988), the elasticity of own-good demand is decomposed using the chain rule as

$$\eta_{q_j,m_j} = \eta_{u,m_j} + \eta_{v_j,m_j} + \eta_{w_j,m_j} \quad \forall j . \tag{14}$$

η_{u,m_j}, η_{v_j,m_j}, and η_{w_j,m_j} are the decision elasticities. η_{v_j,m_j} and η_{w_j,m_j} are own-good decision elasticities because they quantify the impact of a marketing investment in support of good *j* on the conditional choice and conditional quantity decisions about good *j*. I will refer to η_{q_j,m_j} as the comprehensive own-good elasticity.

Gupta's (1988) decision-based decomposition measures the relative influence of changes in consumers' decisions on the increase in own-good demand. The proportions

$$\Lambda_{\text{incidence}, m_j} = \eta_{u,m_j} \big/ \eta_{q_j,m_j} \tag{15}$$

$$\Lambda_{\substack{\text{own-good} \\ \text{choice}, m_j}} = \eta_{v_j,m_j} \big/ \eta_{q_j,m_j} \tag{16}$$

$$\Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}} = \eta_{w_j,m_j} \big/ \eta_{q_j,m_j} \tag{17}$$

summarize the relationship and can be interpreted as follows: Of the growth in own-good demand, $\Lambda_{\text{incidence},\,m_j}$ % is generated by consumers buying more frequently in the category, $\Lambda_{\substack{\text{own-good}\\ \text{choice},\,m_j}}$ % is generated by consumers choosing the target good more frequently when they do buy in the category, and $\Lambda_{\substack{\text{own-good}\\ \text{quantity},\,m_j}}$ % is generated by consumers buying in greater amounts when they do choose the target good.

The influence of changes in consumers' decisions on the demand for competing goods can be quantified in a similar manner. The elasticity of demand for a single competing good is decomposed as

$$\eta_{q_k,m_j} = \eta_{u,m_j} + \eta_{v_k,m_j} + \eta_{w_k,m_j} \quad \forall k \neq j\,. \tag{18}$$

(Proof in appendix.) η_{u,m_j} represents the purchase incidence elasticity. The same term appears in the own-good decomposition, as given in equation (14), because competing goods benefit just like the target good does if consumers buy more frequently in the category and their other decisions are held constant. η_{v_k,m_j} is the elasticity of conditional cross-good choice, and η_{w_k,m_j} is the elasticity of conditional cross-good quantity. Traditionally[4], it has been assumed that the marketing actions of good *j* do not affect the conditional cross-good quantity decisions, which implies $\eta_{w_k,m_j} = 0 \quad \forall k \neq j$. In keeping with this assumption, the elasticity of cross-good demand reduces to

$$\eta_{q_k,m_j} = \eta_{u,m_j} + \eta_{v_k,m_j} \quad \forall k \neq j\,. \tag{19}$$

[4] In making this assumption, Van Heerde et al. (2003, p. 489) note that it is… "used in all five major decomposition articles (Bell et al., 1999; Bucklin et al., 1999; Chiang, 1999; Chintagunta, 1993; Gupta, 1988)."

The elasticities of market and competitive demand can be determined from the elasticities of cross-good demand. Under the assumptions of the demand model,

$$\eta_{Q_{all},m_j} \cdot Q_{all} \quad = \quad \eta_{u,m_j} \cdot Q_{all} \quad + \quad \eta_{w_j,m_j} \cdot q_j \quad + \quad \delta \qquad \text{and} \tag{20}$$

$$\eta_{Q_{-j},m_j} \cdot Q_{-j} \quad = \quad \eta_{u,m_j} \cdot Q_{-j} \quad - \quad \left(\eta_{v_j,m_j} \cdot q_j - \delta\right) \tag{21}$$

where $\delta = \eta_{v_j,m_j} \cdot q_j + \sum_{\substack{k=1 \\ k \neq j}}^{J} \eta_{v_k,m_j} \cdot q_k$.

(Proof in appendix.) The influence of each decision on competitive and market demand can be summarized as follows. *Incidence* – If consumers make purchases more frequently, market and competitive demand increases. *Conditional Quantity* – If consumers buy in greater amounts when they choose the target good, competitive demand remains the same, but market demand increases. *Conditional Choice* – If consumers choose the target good more frequently when they buy in the category, both competitive and market demand can change. As expected, competitive demand decreases. Market demand, however, remains the same ($\delta = 0$) only in the special case that consumers conditionally purchase all goods in the same amounts ($w_j = w \quad \forall j$). If competing goods are purchased in lesser (greater) amounts than the target good, then competitive demand does not decline as much (declines more than) own-good demand increases. The switching offset δ quantifies these changes.[5]

3.2 Empirical Example – van Heerde et al. (2003)

Some confusion still remains about what Gupta's (1988) decision-based

[5] We might be especially concerned about δ in studies that define alternative goods by brand-sizes rather than by brands. For example, market demand grows if consumers switch from buying two 18 oz. boxes of corn flakes to buying three 12 oz. boxes.

decomposition means. To clarify its meaning and to ensure full understanding of its use in section four, let's reconsider his decomposition in the context of van Heerde et al.'s (2003, p. 484) coffee example. Suppose there are 1000 shopping occasions in a given week for coffee. The probability of purchasing coffee on any of these occasions is 0.20, and the conditional probability of choosing Folgers given that coffee is purchased is 0.18. The conditional quantity purchased is 1.0 unit, no matter which brand is chosen. The elasticity of purchase incidence is $\eta_{u,m_j} = 0.034$, of conditional choice is $\eta_{v_j,m_j} = 0.210$, and of conditional quantity is $\eta_{w_j,m_j} = 0.004$ in response to feature-and-display promotion. The comprehensive elasticity of own-good demand is $\eta_{q_j,m_j} = 0.248$.

Van Heerde et al. (2003) incorrectly presume that Gupta's (1988) decomposition holds market demand constant in order to predict the change in competitive demand. They write:

> If we hold category demand constant at 200 units, then under this promotion the non-promoted brands together sell 0.782 * 200 = 156.4 units. This represents a gross decline of 7.6 units from the original sales of 164 units…
>
> Category incidence is not constant, because the incidence probability is now 1.034 * 0.20 = 0.207. This leads to 0.207 *1000 = 207 purchase incidents. According to the model, of the 7 additional purchase incidents, 78.2% should result in the purchase of non-promoted brands, leading to an increase of 0.728 * 7 = 5.4 units. Thus, the net change in sales for non-promoted brands equals -7.6 + 5.4 = -2.2 units (net total sales for the non-promoted brands is 161.8 units).

Applying similar reasoning to the target good, however, leads to a contradiction. The sales of Folgers would increase by 7.6 units if the demand for coffee remained the same. But the demand for coffee would increase, and of the 7 additional purchase incidents, 21.8% would result in purchases of 1.004 units of Folgers, leading to an increase of 0.218 * 7 * 1.004 = 1.5 units. Thus, the change in sales of Folgers would be

7.6 + 1.5 = 9.1 units.[6] This result is problematic, however, because the comprehensive elasticity predicts that the demand for Folgers would grow by 0.248 * 36 = 8.9 units.

Van Heerde et al's (2003) reasoning is incorrect for two reasons. Gupta's (1988) decomposition predicts how the changes in consumers' decisions would affect only own-good demand, not competitive demand. Furthermore, when calculating the influence of any one decision on the growth in own-good demand, Gupta's decomposition holds the consumers' other decisions constant, but does not hold the market demand constant. The proper calculation is based on equation (14). The demand for Folgers would increase by 0.034 * 36 = 1.2 units due to consumers buying coffee more frequently, by 0.210 * 36 = 7.6 units due to consumers choosing Folgers more frequently when they do buy coffee, and by 0.004 * 36 = 0.1 units due to consumers buying coffee in greater amounts when they do buy Folgers. In total, the demand for Folgers would increase by 1.2 + 7.6 + 0.1 = 8.9 units due to the promotion, which reconciles with calculation based on the comprehensive elasticity.

Competitive demand must be decomposed in order to determine how the changes in consumers' decisions would influence the demand for other coffees. Using equation (21), the demand for other coffees would decrease by 0.210 * 36 = 7.6 units due to consumers choosing to buy Folgers more frequently when they do buy coffee. The switching offset would be zero in this example because the conditional purchase quantity is assumed to be the same for all goods ($w_j = 1 \quad \forall j$). But the demand for other coffees would increase by 0.034 * 164 = 5.6 units due to consumers buying coffee more frequently. In total, the demand for other coffees would change by -7.6 + 5.6 = -2.0 units

[6] Making matters somewhat more confusing, van Heerde et al. (2003) state the demand for Folgers increases by 9.2 units. This may be a simple arithmetic mistake.

due to the promotion, not -2.2 units.

Due to this error, a number of the studies that van Heerde et al. (p.483, Table 2) quote as having misinterpreted Gupta's findings actually interpret his findings correctly. For example, they cite Chiang (1991, p. 309) who writes, "These results are similar to the ones obtained by Gupta (1998, p. 352), where 84% of the increase is due to brand switching, 14% by purchase time acceleration, and 2% by increases in quantity." Interpretations, like this one, that suggest Gupta's findings attribute the growth in own-good demand to changes in consumers' decisions are correct.

Other studies quoted by van Heerde et al. (2003) do misinterpret Gupta's findings. For example, they cite Sethuraman and Srinivasan (2002, p.380) who write, "Gupta (1988) and Bell, Chiang and Padmanabhan (1999) show that promotions have a relatively small effect on category expansion compared with brand switching. Therefore, we isolate and study the profitability due to brand switching only." This conclusion cannot be drawn from Gupta's findings. Holding the consumers' other decisions constant, greater purchase frequency can lead to more growth in competitive demand than it would in own-good demand. (In the example, the demand for other coffees would grow by 5.6 units due to greater purchase frequency even though the demand for Folgers would grow only by 1.2 units.) Thus, the market expansion is not necessarily small simply because greater purchase frequency would create a small amount of own-good demand. Combining the decompositions, as will be done in section four, will make the difference between these two interpretations even more distinct.

4. Decompositions that Measure Both Consumer and Competitive Impact

It is possible to measure the consumer and competitive impact of a promotion at the same time. Unified decompositions quantify how a marketing action changes the consumers' decisions of whether, which, and how much to buy and how the change in each of these decisions affects the own-good, competitive, and market demand. I will show how the decision-based decomposition and unit- and share-based decompositions relate to each other and will return to the coffee example to discuss what can be learned from combining them.

4.1 Unifying Relationships

The relationship between the unit- and decision-based decompositions is found by substituting equation (20) into equation (4) and equation (21) into equation (5).

$$\Psi_{\substack{\text{market}\\ \text{expansion, } m_j}} = \frac{1}{s_j} \cdot \Lambda_{\text{incidence, } m_j} + \Lambda_{\substack{\text{own-good}\\ \text{quantity, } m_j}} + \Delta_{\substack{\text{switching}\\ \text{offset, } m_j}} \tag{22}$$

$$\Psi_{\substack{\text{stolen}\\ \text{units, } m_j}} = -\left(\frac{1}{s_j} - 1\right) \cdot \Lambda_{\text{incidence, } m_j} + \Lambda_{\substack{\text{own-good}\\ \text{choice, } m_j}} - \Delta_{\substack{\text{switching}\\ \text{offset, } m_j}}, \tag{23}$$

where $\Delta_{\substack{\text{switching}\\ \text{offset, } m_j}} = \delta \Big/ \left(\eta_{q_j, m_j} \cdot q_j\right)$.

The unit-based measures are functions of the decision-based measures and an additional term, the switching offset. The switching offset accounts for the change in demand due to consumers switching among goods that are conditionally purchased in different amounts.

The relationship between the share- and decision-based decompositions is found by substituting equation (20) into equation (9) and equation (21) into equation (10).

$$\Theta_{\substack{\text{share-preserving}\\ \text{market expansion, } m_j}} = \Lambda_{\text{incidence, } m_j} + s_j \cdot \Lambda_{\substack{\text{own-good}\\ \text{quantity, } m_j}} + s_j \cdot \Delta_{\substack{\text{switching}\\ \text{offset, } m_j}} \tag{24}$$

$$\Theta_{\substack{\text{stolen}\\ \text{share, } m_j}} = S_{-j} \cdot \Lambda_{\substack{\text{own-good}\\ \text{quantity, } m_j}} + \Lambda_{\substack{\text{own-good}\\ \text{choice, } m_j}} - s_j \cdot \Delta_{\substack{\text{switching}\\ \text{offset, } m_j}}. \tag{25}$$

The matrices provided in Table Three express the relationship between the consumer- and competitive-impact decompositions. Panel A depicts the relationship between the decision- and unit-based decompositions and Panel B depicts the relationship between the decision- and share-based decompositions. The cross-good elasticity decomposition is critical to this analysis because it quantifies how changes in each of the consumers' decisions influence competitive and market demand.

< *Insert Table Three* >

The matrices clarify how each of the decompositions accounts for changes in demand. First, it is interesting to note that the information contained in Gupta's (1988) decision-based decomposition is not sufficient to exactly determine the impact of a marketing action on the demand for competing goods. The switching offset needs to be determined in addition to the proportions given in equations (15) to (17) in order to calculate the competitive impact. This is not troubling, however, because the intent of his decomposition is to measure the relative influence of changes in each of the consumers' decision on the growth in own-good demand. An analysis of the impact on cross-good demand, which is required by the competitive-impact decompositions, is not necessary.

Second, strictly speaking, the measure of primary and secondary demand proposed by Bell et al. (1999) can be given neither a unit- nor a share-based interpretation. They define primary demand as the sum of the incidence and conditional own-good quantity proportions, $\Lambda_{\text{incidence},\, m_j}$ and $\Lambda_{\substack{\text{own-good} \\ \text{quantity},\, m_j}}$, and secondary demand as the conditional own-good choice proportion, $\Lambda_{\substack{\text{own-good} \\ \text{choice},\, m_j}}$. While their measure of primary and secondary demand is equivalent to neither of the competitive-impact measures, it does closely approximate share-based measure when two conditions are met: (1) when the proportion of own-good demand that is generated by the conditional own-good quantity decision is small, $\Lambda_{\substack{\text{own-good}\\ \text{quantity}, m_j}} \approx 0$, and (2) when the switching offset is small, $\delta \approx 0$. This approximation may be useful in interpreting previously published results.

Third, it is easy to determine why the general relationship between the decision- and unit-based decompositions is simpler in the special case that the conditional own-good purchase quantity is the same for all goods. Van Heerde et al. (2003) show that if $w_j = w \ \ \forall j$, then

$$\Psi_{\substack{\text{market}\\ \text{expansion}, m_j}} = 1 - \Psi_{\substack{\text{stolen}\\ \text{units}, m_j}} \tag{26}$$

$$\Psi_{\substack{\text{stolen}\\ \text{units}, m_j}} = -\left(\frac{1-v_j}{v_j}\right) \cdot \Lambda_{\text{incidence}, m_j} + \Lambda_{\substack{\text{choice on}\\ \text{own-good}, m_j}} . \tag{27}$$

These relationships hold because the market share of each good is equivalent to its conditional choice probability and the switching offset is zero in this special case. These approximations are useful only when marketing actions have a weak influence on the consumers' conditional own-good quantity decisions, $\eta_{w_j, m_j} \approx 0 \ \ \forall j$ .The conditional own-good quantities are sure to vary across goods, $w_j \neq w \ \ \forall j$, if marketing actions influence these decisions.

4.2 Coffee Example Revisited

Let's revisit the coffee example to further illustrate the relationships between the decompositions. The changes in own-good and competitive demand due to each of the consumers' decisions were determined in section three. The change in market demand, which is also needed, can be constructively determined from equation (20). The demand for coffee would increase by 0.034 * 200 = 6.8 units due to consumers buying coffee more frequently and 0.004 * 36 = 0.1 units due to greater amounts of coffee being purchased when Folgers is chosen. The demand for coffee is unaffected by the consumers' conditional choice decision because the assumptions of the example set the conditional purchase quantities to be equal for all goods. Thus, demand for coffee increases by 6.8 + 0.1 = 6.9 units in total. All of the effects due to the promotion are accounted for in Table Four. Panel A provides the changes in demand by each of the consumers' decisions and Panel B depicts the total changes in demand and market shares.

< *Insert Table Four* >

We are now ready to address the questions that the Folger's brand manager would like to answer, "How will consumers respond and how will competing goods be affected if I choose to invest in feature-and-display advertising?"

The unit- and share-based decompositions measure different aspects of how the promotion affects competing goods. These decompositions depend solely on the total changes in demand for coffee, not on each of the consumers' decisions that give rise to these changes. From the unit-based decomposition, the brand manager learns that of the 8.9 unit growth in demand for Folgers, 77.5% (6.9 units) would be due to an expanded market for coffee and 22.5% (2.0 units) would be due to units being stolen from other coffees. This implies that other coffees would lose 2.0 units due to the promotion. From the share-based decomposition, the brand manager learns that of the 8.9 unit growth in demand for Folgers, 86.5% (7.7 units) would diminish the market share of other coffees.

The decision-based decomposition measures the influence of changes in consumers' decisions on the growth in demand for Folgers. From this decomposition, the brand manager learns that of the 8.9 unit growth in demand for Folgers, 13.5% (1.2 units) would be due to consumers buying coffee more frequently, 85.4% (7.6 units) would be due to consumers choosing Folgers more frequently when they choose to buy coffee, and 1.1% (0.1 units) would be due to consumers purchasing coffee in greater amounts when they choose to buy Folgers. This analysis focuses solely on the growth in demand for Folgers, not on the changes in demand for other coffees.

The unified decompositions, summarized in Table Five, provide a more complete understanding of how the promotion works. It explains why both of the following statements are true: (1) most of the growth in demand for Folgers, 84.5%, is attributed to consumers switching away from other coffees when they choose to buy coffee, but also (2) most of the growth in demand for Folgers, 77.5%, is attributed to market expansion. This paradox exists because, holding the consumers' other decisions constant, a greater frequency of coffee purchases would benefit each brand in proportion to its market share. Folgers, which has a market share of 18%, would earn 1.2 units of the 6.8 unit increase in demand for coffee that is due to greater purchase frequency, but the other coffees would gain 5.6 units, about 4.5 times more. This paradox would not arise if Folgers dominated the market because it would benefit most from a greater frequency of coffee purchases.

< *Insert Table Five* >

Brand managers can use the unified decompositions to help them choose among marketing investments. For instance, suppose that the Folgers' brand manager is particularly concerned about the competitive response to her marketing action. Knowing that a small increase in the consumers' desire for coffee can mitigate much of the losses in competitive demand might persuade her to use an advertising slogan that reads, "Folgers – the best way to brighten your morning!" rather than one that reads "Folgers – the brightest tasting coffee!" It is an empirical question, of course, as to whether the former slogan entices more consumers to drink coffee in the morning. But the unified decompositions provide a means of testing how each of these marketing actions would change consumers' decisions, and, in turn, how the changes in each of these decisions influence own-good and competitive demand.

5. Conclusion

Marketing investments are designed to change consumer behavior in ways that help goods compete in the marketplace. Previous work has focused on how marketing investments affect either consumer decision making or how they affect competing goods. Decision-based decompositions attribute the growth in own-good demand to changes in the consumers' decision-making process. Unit- and share-based decompositions, on the other hand, attribute the same growth to either rivalrous or non-rivalrous sources. Combining the consumer and the competitive points of view in a single decomposition provides a more complete understanding of the marketing investment's impact, which should lead to better managerial decisions. All of the methods that were discussed in this article study a marketing investment's contemporaneous effects. Future work might focus on how the effects of an investment persist over time.

Table One: Units Demanded and Market Shares

	Without Incremental Advertising	With Incremental Advertising	Difference
Firm A	500 units 33.3% share	530 units 34.6% share	+30 units +1.3 share points
Firm B	1000 units 66.7% share	1000 units 65.4% share	No change -1.3 share points
Market Totals	1500 units	1530 units	+30 units

Table Two: Comparison of Unit- and Share-Based Decompositions

Drug	Market Share	Unit-Based Measures		Share-Based Measures	
		Primary Demand	Secondary Demand	Primary Demand	Secondary Demand
Tagamet	25%	0.929	0.071	0.232	0.768
Zantac	75%	0.634	0.366	0.476	0.524

Table Three, Panel A: Relationship between the Unit- and Decision-Based Measures

		Decision-Based Measures: Incidence	Decision-Based Measures: Conditional Quantity	Decision-Based Measures: Conditional Choice	Total
Unit-Based Measures	Market Expansion	$\frac{1}{s_j} \cdot \Lambda_{\text{incidence},\, m_j}$	$\Lambda_{\substack{\text{own-good} \\ \text{quantity},\, m_j}}$	$\Delta_{\substack{\text{switching} \\ \text{offset},\, m_j}}$	$\frac{1}{s_j} \cdot \Lambda_{\text{incidence},\, m_j} + \Lambda_{\substack{\text{own-good} \\ \text{quantity},\, m_j}} + \Delta_{\substack{\text{switching} \\ \text{offset},\, m_j}}$
	Stolen Units	$-\left(\frac{1}{s_j} - 1\right) \cdot \Lambda_{\text{incidence},\, m_j}$	0	$\Lambda_{\substack{\text{own-good} \\ \text{choice},\, m_j}} - \Delta_{\substack{\text{switching} \\ \text{offset},\, m_j}}$	$-\left(\frac{1}{s_j} - 1\right) \cdot \Lambda_{\text{incidence},\, m_j} + \Lambda_{\substack{\text{own-good} \\ \text{choice},\, m_j}} - \Delta_{\substack{\text{switching} \\ \text{offset},\, m_j}}$
	Total	$\Lambda_{\text{incidence},\, m_j}$	$\Lambda_{\substack{\text{own-good} \\ \text{quantity},\, m_j}}$	$\Lambda_{\substack{\text{own-good} \\ \text{choice},\, m_j}}$	

Table Three, Panel B: Relationship between the Share- and Decision-Based Measures

		Decision-Based Measures			
		Incidence	Conditional Quantity	Conditional Choice	Total
Share-Based Measures	Share-Preserving Market Expansion	$\Lambda_{\text{incidence}, m_j}$	$s_j \cdot \Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}}$	$s_j \cdot \Delta_{\substack{\text{switching} \\ \text{offset}, m_j}}$	$\Lambda_{\text{incidence}, m_j} + s_j \cdot \Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}}$ $s_j \cdot \Delta_{\substack{\text{switching} \\ \text{offset}, m_j}}$
	Stolen Share	0	$S_{-j} \cdot \Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}}$	$\Lambda_{\substack{\text{own-good} \\ \text{choice}, m_j}}$ $-s_j \cdot \Delta_{\substack{\text{switching} \\ \text{offset}, m_j}}$	$S_{-j} \cdot \Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}}$ $+\Lambda_{\substack{\text{own-good} \\ \text{choice}, m_j}} - s_j \cdot \Delta_{\substack{\text{switching} \\ \text{offset}, m_j}}$
	Total	$\Lambda_{\text{incidence}, m_j}$	$\Lambda_{\substack{\text{own-good} \\ \text{quantity}, m_j}}$	$\Lambda_{\substack{\text{own-good} \\ \text{choice}, m_j}}$	

Table Four, Panel A: Changes in Demand by Decision

	Purchase Incidence	Conditional Quantity	Conditional Brand Choice	Total
Folgers Coffee	+1.2 units	+0.1 units	+7.6 units	+8.9 units
Other Coffees	+5.6 units	0.0 units	-7.6 units	-2.0 units
All Coffees	+6.8 units	+0.1 units	0.0 units	+6.9 units

Table Four, Panel B: Units Demanded and Market Shares

	Without Promotion	With Promotion	Change
Folgers Coffee	36.0 units	44.9 units	+8.9 units
	18.0% share	21.7% share	+3.7 share points
Other Coffees	164.0 units	162.0 units	-2.0 units
	82.0% share	78.3% share	-3.7 share points
All Coffees	200 units	206.9 units	+6.9 units

Table Five: Measures of Primary and Secondary Demand

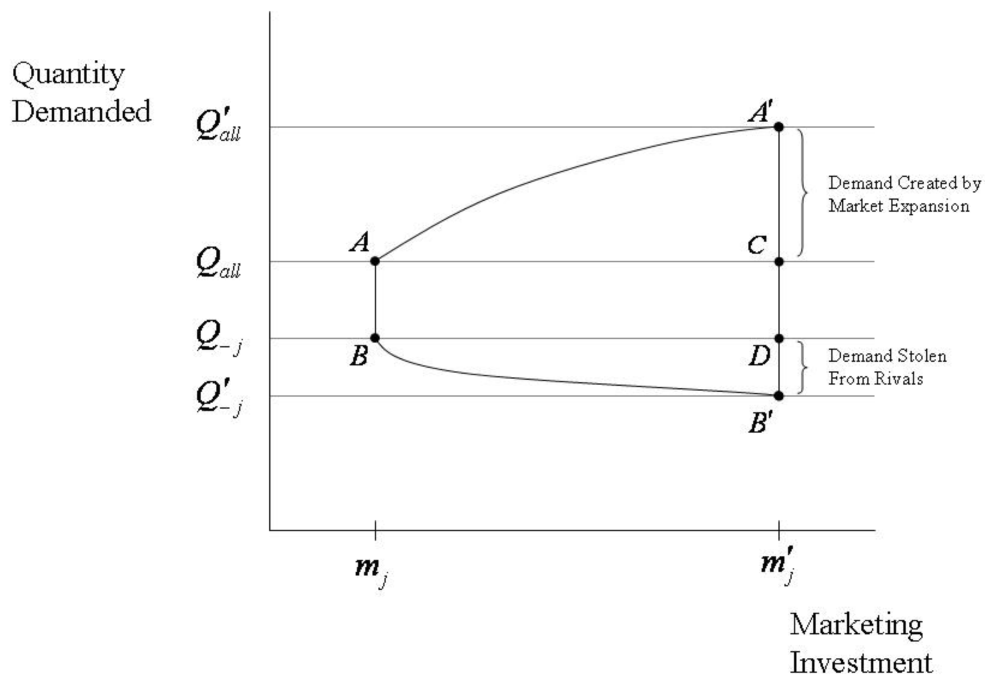
Panel A: Unit- and Decision-Based Decompositions

Source of Change in Demand for Folgers	Incidence (units)	Conditional Quantity (units)	Conditional Choice (units)	Total (units)	Proportional Measures
Market Expansion	+6.8	+0.1	0.0	+6.9	77.5%
Stolen Units	-5.6	0.0	+7.6	+2.0	22.5%
Total (units)	+1.2	+0.1	+7.6	+8.9	
Proportional Measures	13.5%	1.1%	85.4%		

Panel B: Share- and Decision-Based Decompositions

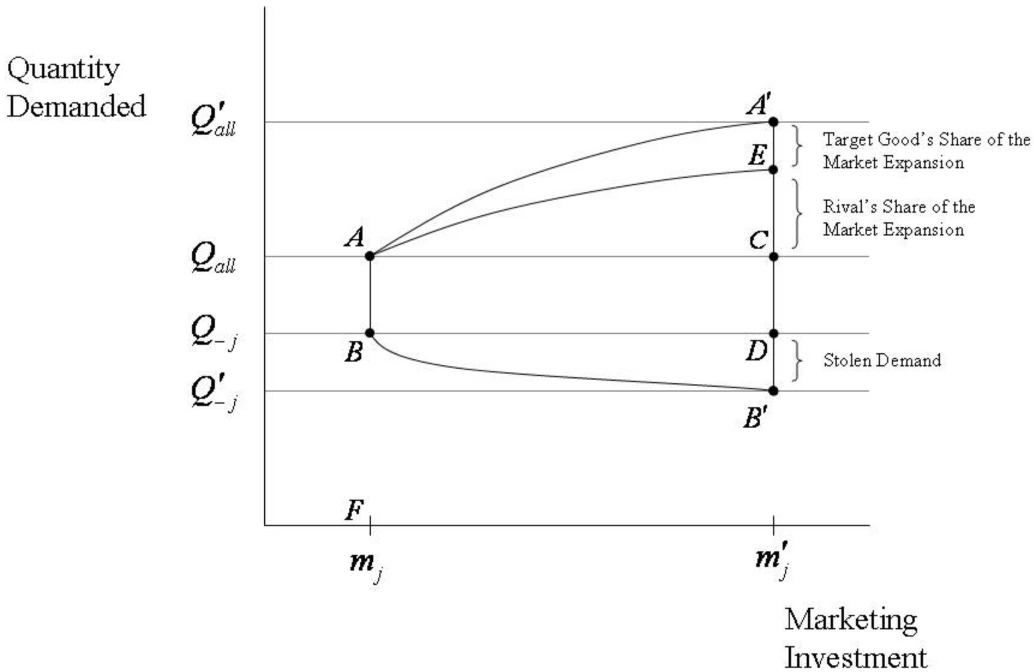
Source of Change in Demand for Folgers	Incidence (units)	Conditional Quantity (units)	Conditional Choice (units)	Total (units)	Proportional Measures
Share-Preserving Market Expansion	+1.2	0.0	0.0	+1.2	13.5%
Stolen Share	0.0	+0.1	+7.6	+7.7	86.5%
Total (units)	+1.2	+0.1	+7.6	+8.9	
Proportional Measures	13.5%	1.1%	85.4%		

Figure One



Quantity
Demanded
Q'_{all}
Q_{all}
Q_{-j}
Q'_{-j}
A'
A
C
B
D
B'
Demand Created by
Market Expansion
Demand Stolen
From Rivals
m_j
m'_j
Marketing
Investment

Figure Two



Quantity Demanded
Q'_{all}
Q_{all}
Q_{-j}
Q'_{-j}
A'
E
A
C
B
D
B'
F
Target Good's Share of the Market Expansion
Rival's Share of the Market Expansion
Stolen Demand
m_j
m'_j
Marketing Investment

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Technical Appendix

Proposition: The change in own-good demand can be decomposed as

$$\frac{\partial q_j}{\partial m_j} = s_j \cdot \frac{\partial Q_{all}}{\partial m_j} - \left[\frac{\partial Q_{-j}}{\partial m_j} + S_{-j} \cdot \frac{\partial Q_{all}}{\partial m_j} \right].$$

Proof:

$$\begin{aligned}
Q_{all} \cdot \frac{\partial s_j}{\partial m_j} &= Q_{all} \cdot \left[\frac{\partial \left(1 - S_{-j}\right)}{\partial m_j} \right] \\
&= Q_{all} \cdot \left[\frac{\partial \left(-Q_{-j} / Q_{all}\right)}{\partial m_j} \right] \\
&= -Q_{all} \cdot \left[\frac{\partial Q_{-j}}{\partial m_j} \cdot \frac{1}{Q_{all}} - \frac{\partial Q_{all}}{\partial m_j} \cdot \frac{Q_{-j}}{Q_{all}{}^2} \right] \\
&= -\left[\frac{\partial Q_{-j}}{\partial m_j} - S_{-j} \cdot \frac{\partial Q_{all}}{\partial m_j} \right]
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial q_j}{\partial m_j} &= s_j \cdot \frac{\partial Q_{all}}{\partial m_j} + Q_{all} \cdot \frac{\partial s_j}{\partial m_j} \\
&= s_j \cdot \frac{\partial Q_{all}}{\partial m_j} - \left[\frac{\partial Q_{-j}}{\partial m_j} + S_{-j} \cdot \frac{\partial Q_{all}}{\partial m_j} \right]
\end{aligned}$$

Q.E.D.

Proposition: Assuming the demand model of equation (13), the cross-good elasticity of demand is $\eta_{q_k,m_j} = \eta_{u,m_j} + \eta_{v_k,m_j} + \eta_{w_k,m_j}$.

Proof:

The demand for good k is

$$q_k = N \cdot u \cdot v_k \cdot w_k \quad for\ k \neq j\ .$$

Applying the chain rule yields

$$\frac{\partial q_k}{\partial m_j} = N \cdot v_k \cdot w_k \cdot \frac{\partial u}{\partial m_j} + N \cdot u \cdot w_k \cdot \frac{\partial v_k}{\partial m_j} + N \cdot u \cdot v_k \cdot \frac{\partial w_k}{\partial m_j}.$$

Thus, the cross-good elasticity is

$$\begin{aligned}
\eta_{q_k,m_j} &= \frac{\partial q_k}{\partial m_j} \cdot \frac{m_j}{q_k} \\
&= \left(N \cdot v_k \cdot w_k \cdot \frac{\partial u}{\partial m_j} + N \cdot u \cdot w_k \cdot \frac{\partial v_k}{\partial m_j} + N \cdot u \cdot v_k \cdot \frac{\partial w_k}{\partial m_j} \right) \cdot \frac{m_j}{q_k} \\
&= \frac{\partial u}{\partial m_j} \cdot \frac{m_j}{u} + \frac{\partial v_k}{\partial m_j} \cdot \frac{m_j}{v_k} + \frac{\partial w_k}{\partial m_j} \cdot \frac{m_j}{w_k} \\
&= \eta_{u,m_j} + \eta_{v_k,m_j} + \eta_{w_k,m_j}
\end{aligned}$$

Q.E.D.

Proposition: Assuming the demand model of equation (13) and that $\eta_{w_k,m_j} = 0 \quad \forall k \neq j$, $\eta_{Q_{all},m_j} \cdot Q_{all} = \eta_{u,m_j} \cdot Q_{all} + \eta_{w_j,m_j} \cdot q_j + \delta$.

Proof:

$$\begin{aligned}
\eta_{Q_{all},m_j} \cdot Q_{all} &= \sum_{k=1}^{J} \eta_{q_k,m_j} \cdot q_k \\
&= \sum_{k=1}^{J} \left[\eta_{u,m_j} + \eta_{v_k,m_j} + \eta_{w_k,m_j} \right] \cdot q_k
\end{aligned}$$

$$= \eta_{u,m_j} \cdot Q_{all} + \eta_{w_k,m_j} \cdot q_j + \sum_{k=1}^{J} \eta_{v_k,m_j} \cdot q_k$$

$$= \eta_{u,m_j} \cdot Q_{all} + \eta_{w_k,m_j} \cdot q_j + \delta$$

where $\delta = \eta_{v_j,m_j} \cdot q_j + \sum_{\substack{k=1 \\ k \neq j}}^{J} \eta_{v_k,m_j} \cdot q_k$

Q.E.D.

Proposition: Assuming the demand model of equation (13) and that $\eta_{w_k,m_j} = 0 \quad \forall k \neq j$, $\eta_{Q_{-j},m_j} \cdot Q_{-j} = \eta_{u,m_j} \cdot Q_{-j} - \left(\eta_{v_j,m_j} \cdot q_j - \delta\right)$.

Proof:

$$\eta_{Q_{-j},m_j} \cdot Q_{-j} = \sum_{\substack{k=1 \\ k \neq j}}^{J} \eta_{q_k,m_j} \cdot q_k$$

$$= \sum_{\substack{k=1 \\ k \neq j}}^{J} \left[\eta_{u,m_j} + \eta_{v_k,m_j} + \eta_{w_k,m_j}\right] \cdot q_k$$

Assuming that $\eta_{w_k,m_j} = 0 \quad \forall k \neq j$,

$$\eta_{Q_{-j},m_j} \cdot Q_{-j} = \eta_{u,m_j} \cdot Q_{-j} + \sum_{\substack{k=1 \\ k \neq j}}^{J} \eta_{v_k,m_j} \cdot q_k$$

$$= \eta_{u,m_j} \cdot Q_{-j} - \eta_{v_j,m_j} \cdot q_j + \delta$$

where $\delta = \eta_{v_j,m_j} \cdot q_j + \sum_{\substack{k=1 \\ k \neq j}}^{J} \eta_{v_k,m_j} \cdot q_k$

Q.E.D.