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Why Do Intermediaries Divert Search? – Companion Paper

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In this companion to our main paper we provide several extensions of the model developed there.

1 Revenues from consumer traffic

Suppose that in addition to revenues r_i from the transactions conducted by consumers at stores, the intermediary also receives revenues t for each consumer who uses the intermediary's service. $t > 0$ can be interpreted as a revenue coming from indirect sources, such as advertisers paying for the privilege of reaching the consumer audience offered by the intermediary.[1] $t < 0$ can be interpreted as a cost incurred by the intermediary for serving each consumer.[2] The expression of the intermediary's revenues becomes:

$$(\widetilde{r_1} + \widetilde{r_2}) F\left(u^L\right) + \left[\widetilde{r_1} + (1-p)\widetilde{r_2}\right]\left[F\left(u\left(q\right)\right) - F\left(u^L\right)\right] + tF\left(u\left(q\right)\right)$$

where $u\left(q\right) = \frac{u^H + (1-q)u^L}{2-q}$

The condition for some search diversion to be optimal is now:

$$\frac{\widetilde{r_1} + t}{\widetilde{r_2}} < \frac{F\left(u^H\right) - F\left(u^L\right)}{f\left(u^H\right)\left(u^H - u^L\right)}$$

Thus, the intermediary is unambiguously *less* likely to divert search when t is higher, i.e. when it extracts more revenues from consumer traffic, which is quite intuitive. Also, it is easily verified that the slope of the profit function with respect to q is increasing in t, implying that the optimal level of search diversion q^* is increasing in t.

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[1] This is customary practice with shopping malls, who oftentimes rent out various parts of their space to companies wishing to showcase their products (e.g. cars).

[2] Again, in the case of shopping malls, this cost may cover: printing maps, maintaining the cleanliness of the complex, etc.

2 Substitutability/complementarity between stores

We are now interested in the effect of relaxing the assumption that consumers view stores as independent (no complementarity/substitutability among them) on the optimal level of search diversion chosen by the intermediary. Specifically, substitutability (complementarity) is modeled by assuming that *conditional* on already having enjoyed utility u^i, the utility u^j is reduced (respectively increased) from u^j to $u^j - \gamma$ (respectively $u^j + \gamma$), where $\gamma > 0$, where $j \neq i \in \{L, H\}$.

2.1 Substitutability

In this case, consumers with $c \leq u^L - \gamma < u^H - \gamma$ shop at both stores no matter what.

Consumers with $u^L - \gamma \leq c \leq u^H - \gamma$ only shop at their less preferred store if they are diverted there while looking for their favorite store. Their net utility is $qu^H + (1-q)\left(u^L + u^H - \gamma\right) - (2-q)\,c$ and is positive if and only if:

$$c \leq \frac{u^H + (1-q)\left(u^L - \gamma\right)}{2-q}$$

Finally, consumers with $u^H \geq c \geq u^H - \gamma$ shop at most at one store and then stop. Therefore, they visit the platform if and only if:

$$c \leq qu^H + (1-q)\,u^L$$

Note that:

$$\frac{u^H + (1-q)\left(u^L - \gamma\right)}{2-q} \geq u^H - \gamma \Longleftrightarrow \gamma \geq \left(u^H - u^L\right)(1-q)$$

and:

$$qu^H + (1-q)\,u^L \geq u^H - \gamma \Longleftrightarrow \gamma \geq \left(u^H - u^L\right)(1-q)$$

Thus, we obtain the expression of platform profits:

$$\Pi^P = \begin{cases} \begin{array}{l} (\widetilde{r_1} + \widetilde{r_2})\,F\left(u^L - \gamma\right) \\ + (\widetilde{r_1} + (1-q)\,\widetilde{r_2})\left[F\left(\frac{u^H + (1-q)\left(u^L - \gamma\right)}{2-q}\right) - F\left(u^L - \gamma\right)\right] \end{array} & \text{if } \ \gamma \leq \left(u^H - u^L\right)(1-q) \\ \\ \begin{array}{l} (\widetilde{r_1} + \widetilde{r_2})\,F\left(u^L - \gamma\right) \\ + (\widetilde{r_1} + (1-q)\,\widetilde{r_2})\left[F\left(u^H - \gamma\right) - F\left(u^L - \gamma\right)\right] \\ + (q\widetilde{r_1} + (1-q)\,\widetilde{r_2})\left[F\left(qu^H + (1-q)\,u^L\right) - F\left(u^H - \gamma\right)\right] \end{array} & \text{if } \ \gamma \geq \left(u^H - u^L\right)(1-q) \end{cases}$$

We are interested in the conditions under which the intermediary will divert search, i.e. choose $q^* < 1$. Given $\gamma > 0$, for q close enough to 1 we will eventually have $\gamma \geq \left(u^H - u^L\right)(1-q)$, therefore if the derivative of the second expression above evaluated at $q = 1$ is negative, we can conclude that the optimal q is smaller than 1. This condition is equivalent to:

$$H\left(\widetilde{r_1},\widetilde{r_2},\gamma\right)\equiv\frac{\widetilde{r_1}}{\widetilde{r_2}}\left(1+\frac{F\left(u^H\right)-F\left(u^H-\gamma\right)}{\left(u^H-u^L\right)f\left(u^H\right)}\right)-\frac{F\left(u^H\right)-F\left(u^L-\gamma\right)}{\left(u^H-u^L\right)f\left(u^H\right)}\leq 0$$

For $\gamma=0$ we obtain condition (5) derived in the main paper. Here we are interested in knowing whether introducing a small amount of substitutability makes degradation of quality more or less likely. In order to determine this, note that:

$$\frac{\partial H}{\partial\gamma}\left(\widetilde{r_1},\widetilde{r_2},\gamma=0\right)<0\Longleftrightarrow\frac{\widetilde{r_1}}{\widetilde{r_2}}<\frac{f\left(u^L\right)}{f\left(u^H\right)}$$

If F is concave, we have $\frac{f\left(u^L\right)}{f\left(u^H\right)}>\frac{F\left(u^H\right)-F\left(u^L\right)}{\left(u^H-u^L\right)f\left(u^H\right)}$ and therefore we obtain:

Proposition C1 *When F is concave, introducing an arbitrarily small amount of substitutability between the two stores ($\gamma\to 0$) makes it* more *likely that the optimal level of search effectiveness q will be less than 1.*

■

2.2 Complementarity

Suppose now that stores are complementary, i.e. conditional on having visited store i, the utility from visiting store $j\neq i$ is increased from u_j to $u_j+\gamma$, where $\gamma>0$.

It is then easily shown that:

$$\Pi^P=\begin{cases}\begin{array}{l}\left(\widetilde{r_1}+\widetilde{r_2}\right)F\left(u^L+\gamma\right)\\+\left(\widetilde{r_1}+(1-q)\widetilde{r_2}\right)\left[F\left(\frac{u^H+(1-q)\left(u^L+\gamma\right)}{2-q}\right)-F\left(u^L+\gamma\right)\right]\end{array} & \text{if}\quad\gamma\leq u^H-u^L\\[2em]\left(\widetilde{r_1}+\widetilde{r_2}\right)F\left(u^L+\gamma\right) & \text{if}\quad\gamma\geq u^H-u^L\end{cases}$$

When γ is small, we are in the first scenario. Then the intermediary diverts search if and only if:

$$\frac{\widetilde{r_1}}{\widetilde{r_2}}\leq\frac{F\left(u^H\right)-F\left(u^L+\gamma\right)}{\left(u^H-u^L-\gamma\right)f\left(u^H\right)}$$

For $\gamma=0$ we obtain (5) in the main paper. If F is concave then $\frac{F\left(u^H\right)-F\left(u^L+\gamma\right)}{\left(u^H-u^L-\gamma\right)f\left(u^H\right)}$ is decreasing, therefore, for small enough γ, complementarity between the stores makes search diversion *less* likely.

Proposition C2 *When F is concave, introducing an arbitrarily small amount of complementarity between the two stores makes it* less *likely that the optimal level of search effectiveness q will be less than 1.*

■

The interpretation of the results contained in Propositions C1 and C2 is straightforward: complementarity makes it less necessary to divert consumers in order to convince them to visit their less preferred store (the attractiveness of that store conditional on having visited their favorite store), while substitutability makes it more necessary.

3 Independent values

Another variation of our basic model consists in assuming that the two stores are *ex-ante* identical and there is only one type of consumers, each of whom can have valuation $v = u^H$ or $v = u^L$ for each store. The valuations for the two stores are independent across consumers and across stores for the same consumer. For any consumer and any of the two stores, denote $x = prob\left(v = u^H\right)$ the *ex-ante* probability that a given store yields utility u^H to a given consumer.

Whenever a consumer comes to the intermediary, her valuations are revealed to the intermediary but not known by the consumer. If a consumer's valuations are (u^H, u^L) or $\left(u^L, u^H\right)$ for store 1 and store 2 respectively, then the intermediary directs her to the store for which she has valuation u^H with probability q in the first search. If her valuations are (u^i, u^i) for $i = L$ or H then the intermediary directs her to a store for which she has valuation u^i with probability 1. As before, the intermediary commits to q first and q is observed by consumers before deciding whether or not to visit. Finally, the intermediary derives revenues $r_1 = r_2 = r$ per consumer visit at either store.

One interpretation of this version of the basic model is a recommendation system which only relies on customer-specific specific data. Upon arriving at the intermediary, neither the consumer nor the intermediary know the match between the consumer's preferences and the stores (they do agree on the prior probability x) but the platform can infer it based on some consumer-specific information that it observes (e.g. purchase history).

Denote by $U_k(q)$ the expected utility of a consumer's second search if the first yields u^k, $k = H, L$. Then:

$$\begin{aligned}
U_L(q) &= \frac{2x(1-x)(1-q)}{(1-x)^2 + 2x(1-x)(1-q)}\left(u^H - u^L\right) + u^L \\
U_H(q) &= \frac{x^2}{x^2 + 2x(1-x)q}\left(u^H - u^L\right) + u^L > U_L(q)
\end{aligned}$$

Also, let:

$$Eu = xu^H + (1-x)u^L$$

Note that $U_H(q) < Eu$ for $q > \frac{1}{2}$, which we will assume from now on. We have:

Lemma C1 *$U_L(q)$ and $U_H(q)$ are decreasing in q and $U_H(q) > U_L(q)$ for all q.*

Proof Straightforward.

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The expected utility from going through the intermediary for a consumer with search cost c is:

$$\begin{aligned} V_{INT}(c) &= \left(x^2 + 2x(1-x)q\right)\left(u^H + \max\left(U_H(q) - c, 0\right)\right) \\ &\quad + \left((1-x)^2 + 2x(1-x)(1-q)\right)\left(u^L + \max\left(U_L(q) - c, 0\right)\right) - c \end{aligned} \tag{1}$$

By contrast, if the consumer decides to do away with the intermediary and visit the stores without taking any advice from it, her expected utility is:

$$V_{DIY}(c) = xu^H + (1-x)u^L + \max\left(0, xu^H + (1-x)u^L - c\right) - c$$

which is equal to:

$$V_{DIY}(c) = \max\left(0, 2(Eu - c)\right) \tag{2}$$

Let:

$$\begin{aligned} Y(q) &\equiv \left(x^2 + 2x(1-x)q\right)\left(u^H - u^L\right) + u^L \\ &= Eu + \left(u^H - u^L\right)x(1-x)(2q-1) > Eu \end{aligned}$$

We then have:

Lemma C2 *A consumer with search cost c heeds the intermediary's recommendation and visits at least one store if and only if $c \leq Y(q)$. Consumers with $c > Y(q)$ do not go through to the intermediary and do not visit any store.*

Proof Assume first $c \leq U_L(q)$. Then:

$$V_{INT}(c) - V_{DIY}(c) = \left(2x^2 + 2x(1-x)\right)\left(u^H - u^L\right) + 2u^L - 2x\left(u^H - u^L\right) - 2u^L = 0$$

Thus consumers with low search cost are indifferent between searching by themselves and going through the intermediary (regardless of the probability q). They always visit both stores.

Assume now $U_L(q) \leq c \leq U_H(q)$. In this case $V_{INT}(c) \geq V_{DIY}(c)$ if and only if:

$$\frac{c - u^L}{u^H - u^L} \geq \frac{2x(1-q)}{1 - x + 2x(1-q)}$$

But $U_L(q) \leq c$ is equivalent to the same condition hence the inequality above is satisfied. These consumers visit one store and continue if and only if they encounter an H store upon the first search.

Third, assume $U_H(q) \leq c \leq Eu$. In this case $V_{INT}(c) \geq V_{DIY}(c)$ if and only if:

$$\frac{c - u^L}{u^H - u^L} \geq 2x - x^2 - 2x(1-x)q$$

Meanwhile, $U_H(q) \leq c$ is equivalent to:

$$\frac{c-u^L}{u^H-u^L} \geq \frac{x^2}{x^2+2x(1-x)q}$$

It is then easily shown that for all q:

$$\frac{x^2}{x^2+2x(1-x)q} \geq 2x - x^2 - 2x(1-x)q$$

which implies that $V_{INT}(c) \geq V_{DIY}(c)$ on this interval too. These consumers only visit one store.

The last possible case is $c \geq Eu$ or:

$$\frac{c-u^L}{u^H-u^L} \geq x$$

Consumers with search costs verifying this inequality will never visit stores by themselves:

$$V_{DIY}(c) = 0$$

On the other hand, the condition $V_{INT}(c) \geq 0$ can now be written:

$$\frac{c-u^L}{u^H-u^L} \leq x^2 + 2x(1-x)q$$

Note that:

$$x^2 + 2x(1-x)q > x$$

for $q > \frac{1}{2}$ so that the marginal consumer which visits the intermediary is $Y(q)$.

$\blacksquare$

We can now derive the expression of the intermediary's profits:

$$\Pi^I(q) = F(U_L(q)) \times 2r + [F(U_H(q)) - F(U_L(q))] \times \left(1 + x^2 + 2x(1-x)q\right) r + [F(Y(q)) - F(U_H(q))] \times r$$

yielding:

$$\frac{\Pi^I(q)}{r} = F(Y(q)) + F(U_H(q))\left(x^2+2x(1-x)q\right) + F(U_L(q))\left((1-x)^2 + 2x(1-x)(1-q)\right)$$

Straightforward calculations lead to:

Proposition C3 *The intermediary diverts search* $\frac{\partial \Pi^P}{\partial q}(q=1) < 0$ *if and only if:*

$$f(Y(1)) + \frac{F(U_H(1)) - F(U_L(1))}{u^H-u^L} < f(U_H(1))\frac{x}{2-x} + f(U_L(1))$$

$\blacksquare$

With x very close to 1, we have $Y(1) \sim u^H$, $U_H(1) \sim u^H$ and $U_L(1) \sim u^L$. The condition above then becomes:

$$1 > \frac{F\left(u^H\right) - F\left(u^L\right)}{f\left(u^L\right)\left(u^H - u^L\right)}$$

which is exactly the same as (5) in the main paper when $r_1 = r_2$. (Note that this condition is satisfied when F is concave.)

4 Endogenous store prices: per click vs. per sales royalties

In section 3.2 of the main paper (where stores set prices individually in response to q chosen by the intermediary), we assumed that the intermediary charged stores per-click royalties. Here we prove that the main conclusion of that section (Proposition 3) is unchanged if instead the intermediary charges per-sales royalties.

With exogenously fixed per sales royalties ρ, the expression of store profits (8) becomes:

$$\Pi_i^S(p_i, p_j, q) = (1-\rho)\left\{ R^H(p_i) \underbrace{\frac{1}{2}F\left(u(p_i, p_j, q)\right)}_{\substack{\text{store } i\text{'s traffic by} \\ \text{type } i \text{ consumers}}} + R^L(p_i)\underbrace{\frac{1}{2}\left[qF\left(u^L(p_i)\right) + (1-q)F\left(u(p_j, p_i, q)\right)\right]}_{\substack{\text{store } i\text{'s traffic by} \\ \text{type } j \text{ consumers}}} \right\}$$

where $u(p_i, p_j, q) \equiv \frac{u^H(p_i) + (1-q)u^L(p_j)}{2-q}$.

Thus, when both stores charge the same price p (which is the case in the symmetric equilibrium) the intermediary's profits are given by:

$$\rho\left\{R^H(p)F\left(\frac{u^H(p) + (1-q)u^L(p)}{2-q}\right) + R^L(p)\left[qF\left(u^L(p)\right) + (1-q)F\left(\frac{u^H(p) + (1-q)u^L(p)}{2-q}\right)\right]\right\}$$

First, note that if both store prices were fixed at p, then the intermediary diverts search if and only if:

$$\frac{R^H(p)}{R^L(p)} \leq \frac{F\left(u^H(p)\right) - F\left(u^L(p)\right)}{\left(u^H(p) - u^L(p)\right) f\left(u^H(p)\right)}$$

Comparing this condition with (10) in the main paper and recalling that $R^H(p) > R^L(p)$, we have:

Proposition C4 *With fixed store prices, all other things being equal, an intermediary charging per-click fees is more likely to divert search than an intermediary charging per sales fees.*

■

This result has a simple interpretation: per-sales charges make the intermediary's interests more aligned with those of the stores, therefore it does not need to divert search as much.

If stores can individually choose their prices but consumers do not observe these prices prior to visiting stores, then store i's profits are:

$$(1-\rho)\left\{R^H(p_i)\frac{1}{2}F(u(p^e,p^e,q))+R^L(p_i)\frac{1}{2}\left[qF\left(u^L(p^e)\right)+(1-q)F(u(p^e,p^e,q))\right]\right\}$$

where p^e is the symmetric store price equilibrium expected by consumers.

Then the equilibrium price $p^*(q)$ is again (just like in the case with per-click royalties treated in the main paper) the solution to:

$$p^*=\arg\max_p\left\{R^H(p)F(u(p^*,p^*,q))+R^L(p)\left[qF\left(u^L(p^*)\right)+(1-q)F(u(p^*,p^*,q))\right]\right\} \tag{3}$$

Lemma 1 from the main paper still applies, therefore $\frac{dp^*}{dq}>0$.

We can now calculate:

$$\frac{\partial\Pi^I}{\partial p}(q=1)=\rho\left[\begin{array}{c}\frac{dR^H}{dp}(p^*(1))F\left(u^H(p^*(1))\right)+\frac{dR^L}{dp}(p^*(1))F\left(u^L(p^*(1))\right)\\+R^H\frac{du^H}{dp}(p^*(1))f\left(u^H(p^*(1))\right)+R^L\frac{du^L}{dp}(p^*(1))f\left(u^L(p^*(1))\right)\end{array}\right]$$

But (3) implies:

$$\frac{dR^H}{dp}(p^*(1))F\left(u^H(p^*(1))\right)+\frac{dR^L}{dp}(p^*(1))F\left(u^L(p^*(1))\right)=0$$

therefore:

$$\frac{\partial\Pi^I}{\partial p}(q=1)=\rho\left[R^H\frac{du^H}{dp}(p^*(1))f\left(u^H(p^*(1))\right)+R^L\frac{du^L}{dp}(p^*(1))f\left(u^L(p^*(1))\right)\right]<0$$

Thus, the conclusion in Proposition 3 from the main paper remains unchanged: when store prices are endogenous, the intermediary has an incentive to lower the level of search effectiveness further than when store prices are exogenously given in order to induce lower store prices.