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## Incentive theory with data compression

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*Jerry R. Green and Jean-Jacques Laffont*

### I Introduction

The economics of information and of incentives has been one of the most active areas of research in economic theory over the past fifteen years. The origin of this work can be traced to the writings of Marschak in the 1920s and, beyond that, to von Hayek and other European authors. During the development of this theory, the nature of the problems studied has steadily evolved. The kinds of informational difficulties in complex organizations that captured the attention of early writers were actually far different from the questions to which modern economic theory has provided interesting answers.

A brief digression on the nature of this evolution will help to define our interests more clearly. In the earliest papers, the issue was the design and evaluation of communication processes. The economy was depicted as continually changing. A good system of communication was one that could quickly and accurately disseminate information about its current state. Writers such as von Hayek (1945) wanted to evaluate the price system in this informational role. Emphasis was primarily on the continued *flow* of new information, and on the transitory character of the state of the economy.

The sort of knowledge with which I have been concerned is knowledge of the kind which by its nature cannot enter into statistics and therefore cannot be conveyed to any central authority in statistical form. (von Hayek, 1945, p. 524)

... an essential part of the phenomena with which we have to deal: the unavoidable imperfection of man's knowledge and the consequent need of a process by which knowledge is constantly communicated and acquired. (von Hayek, 1945, p. 530)

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The next stage in the economics of information centered on comparing communication networks. This led naturally to the theory of teams, a subject that was almost twenty years in gestation until the publication of the path-breaking work by Marschak and Radner (1972).<sup>1</sup> The theory of teams dropped the concepts of information flow about a continually changing state in favor of a more static view, with the exception of Chapter 7 in Marschak and Radner (1972). The state of the system was fixed, and team members each possessed different information about it. This was in the tradition of statistical decision theory, where one regards the parameters of a distribution as unknown, and seeks optimal responses to the available information. Marschak and Radner introduced the concept of a team decision rule. One can say that team theory *is* multiplayer statistical decision theory. They tried to study the question of optimal communication structure within the team. However, it is fair to say that most of their results concern the characterization of the optimal decision rule for a fixed communication structure, rather than this comparison.

The final stage, within the last ten years, involved the explicit introduction of differences in objectives among the agents. Communication problems of a technological nature were largely ignored. Instead, most attention was directed at the problem of providing the incentives necessary to make the self-interested agents divulge their information. The idea of a communication network was no longer pursued.

One reason for the rapid acceleration of this type of economic theory is the wide applicability it has found in other areas of economics. Problems of optimal taxation, sorting and screening, adverse selection in insurance, employment contracting, the theory of auctions, among others, have all been shown to be special cases of the general information transmission problem.

To recapitulate this development, we have seen that the ideas of information as a flow and of communication as a complex network design problem have been replaced by a static view of the state of the system and a costless technology for information transmission. This chapter reintroduces the idea of information as a flow to study the interaction of communication constraints and incentives questions.

Section II describes informally the type of principal-agent problem on which we focus. Section III shows how information theory is used to formalize the problem. The solution to the principal-agent problem is given in Section IV. Section V compares the results to those that would be obtained by a social planner.

## II A principal-agent problem with information flows

We consider a two-member organization consisting of a principal and an agent. The agent observes a sequence of realizations of a random process,  $x_t$ , and attempts to control a sequence of decisions,  $y_t$ . The actual decisions are taken by an obedient subordinate of the agent. The problem is that the agent cannot communicate perfectly with the subordinate because the channel through which this communication is to flow cannot accommodate all the information about  $x_t$ . The details of the communication technology will be discussed below. The agent and subordinate use the communication technology as effectively as possible, so as to optimize a common objective: minimizing an expected loss function that depends on  $x_t$  and  $y_t$ . This objective differs from that of the principal. The principal's only role in this model is to provide and pay for the communication technology for the agent and subordinate. The principal knows that, whatever technology is provided, the agent and subordinate will use it to their best advantage and that this may be different from the principal's.

This chapter studies how the resources devoted by the principal to the communication technology depend on the difference in the objectives and on the characteristics of the source  $x_t$ . We compare the amount of resources devoted to communication in this system with those that would be used by a principal who could observe  $x_t$  and who could communicate directly to the subordinate. We also compare it to the optimal system, as it would be designed by a social planner to maximize the joint welfare of the principal and the agent, subject to the fact that the agent will observe  $x_t$  and will control the use of this communication channel.

The novel feature of this system is the way in which the agent's environment is modeled, particularly the dynamic aspects of his receipt and transmission of information. As this is quite distinct from the usual principal-agent framework, we will now describe it in some detail.

Imagine that the observations  $x_t$  are realizations of independent identically distributed random variables taking one of  $M$  possible values, and that these realizations are perceived by the agent at the rate of  $S_0$  per unit time. For example,  $x_t$  could be the rate of new orders placed at different locations in a distribution system for some manufactured product. Each order consists of a quantity and a place of delivery. Both components arrive at random and are expressed in the message  $x_t$ .

The agent has to transmit information about  $x_t$  to a subordinate. This is modeled by the agent's ability to send a sequence of pieces of informa-

tion  $z_\tau$  to the subordinate. The frequency with which this information  $z_\tau$  can be transmitted may be different from the rate at which the random inputs  $x_t$  arrive. It is for this reason that we denote the times at which the inputs arrive by  $t$  and the time at which messages are sent by  $\tau$ . For example, if inputs arrive each day, but messages can be sent each hour,  $t = 1, 2, \dots$ , but  $\tau = \frac{1}{24}, \frac{2}{24}, \dots$ . Typically, we imagine that messages are sent at a rate,  $S_1$  per unit time, substantially higher than  $S_0$ , but that each  $z_\tau$  lies in a set with  $L$  distinct points, where  $L$  is much smaller than  $M$ . For example, the transmission might be in Morse code or certain abbreviations and approximations might be used. These qualitative relations are not relevant to the nature of our results. The important point is that the information system may not be able to fully and accurately transmit all the inputs. Information theory gives us one way to model this inability precisely.

The agent's subordinate, having received the sequence  $z_\tau$ , decodes it as well as possible. The subordinate would, ideally, reconstruct  $x_t$  and act upon it. However, unless  $S_1$  is large enough,  $z_\tau$  will not contain enough information to reconstruct  $x_t$  perfectly. Agent and subordinates have the ability to arrange any system of encoding and decoding they desire. Let the best reconstruction of  $x_t$  be denoted  $y_t$ . The result is that the subordinate will act according to  $y_t$ .

We now describe the optimal encoding and decoding further, pursuing the example of orders for delivery of a manufactured product described above. Suppose that the communication technology allows the place of the order to be reconstructed with perfect accuracy, but that the level of the order can only be reconstructed to within 1,000 units. Any more accurate reconstruction of orders would require more transmissions of  $z_\tau$  per unit time than the channel could handle. Were the agent and subordinate to attempt this with the current channel, it would take too much time and unfilled orders would pile up without bound. We assume that a fixed delay between  $x_t$  and  $y_t$ , due to the time taken in the encoding, transmission, and decoding process, can be tolerated without loss. But orders must be processed *at the same rate* as they arrive.

The essential difference between this model and the usual principal-agent models can now be described. The most important difference is the idea that the information received by the agent is arriving as a flow over time. It is this idea that allows us to treat the communication technology available to the agent as a continuous variable – the rate per unit time at which messages can be sent.

In the usual principal-agent model, the agent receives some informa-

tion and is allowed to act on it at will, perhaps subject to a limitation on the set of possible actions. In this static context, the literature on the principal-agent problem has modeled the difficulties encountered by the agent in actually implementing his desired choice by employing a *dimension counting* approach. The random variable  $x_i$  lies in a space of fixed dimension, the agent's action is constrained to span a space of lower dimension, but the selection of this action is still in the hands of the agent.<sup>2</sup> Such an approach is technically very messy, and its "integer" nature makes it hard to study by analytical means the principal's choice of a space to provide for the agent. In environments where enough informational resources exist to achieve a fully efficient economic allocation, Mount and Reiter (1974) and Reichelstein (1983) have characterized the minimum dimensionality of the space in which messages must be allowed to vary. At present, a "second-best" approach to these problems along "dimensionality" lines – that is, asking how many dimensions are necessary to achieve a given, suboptimal, efficiency criterion – has not been attempted. This chapter addresses such second-best issues by modeling informational resources as the continuous variable speed of transmission available between the agent and subordinate, instead of the integer-valued dimensionality measure.

### III Detailed model of agent's behavior and information theory

The precise model of the agent's behavior that we will use below needs some justification and explanation. In the process of offering it, we will explain some related ideas in information theory.

As described above, the agent receives  $x_i$  and can transmit  $z_i$ . The way in which  $x_i$  should be coded into  $z_i$  depends on the statistical distribution of  $x_i$  and on the ratio of the speed of input arrival to the speed of the transmission. For example, in English, certain letters such as  $q$  and  $x$  are very rare relative to  $e$  and  $s$ . Therefore their Morse code symbols are longer strings of dots and dashes – four such symbols instead of one or two. If the letters in English were arriving at the rate of one per second, and were encoded in Morse code, we would need to send about 2.4 Morse code symbols per second to transmit at the required rate. If  $S_1/S_0$  were below 2.4, we would either have to tolerate some mistakes or we would have to find a more efficient code. Mistakes might be, for instance, that certain rarely used letters could be assigned the same code. Thus,  $q$  and  $x$  might not be distinguishable upon decoding. Hopefully this would not cause too much of a loss.

One of the main ideas of information theory has been to find the most

efficient method for encoding. It has asked whether, for a given transmission speed, some code can be found that will allow for error-free decoding. The solution to this problem is to code long strings of letters into long "code words." For example, instead of coding each letter in English into a Morse code sequence of from one to four letters, longer sequences of input letters, say ten at a time, might be encoded into longer strings of dots and dashes, say strings of ten to twenty such symbols. Such codes are called block codes.

One of the most powerful results of information theory<sup>3</sup> concerns the maximal amount of compression that can be achieved as more and more complex block codes are allowed. For any  $\epsilon > 0$ , perfect encoding, transmission, and decoding of a stationary source,  $x_t$  sending  $S_0$  characters of the source per second, is possible using a language with  $L$  symbols, if the speed of transmission of  $z_t$  is  $S_0 H + \epsilon$  where  $H$  is the entropy of the source (log base  $L$ ) and if a sufficiently long block-code is used. Conversely, if  $S_1 < S_0 H$ , then some error must be tolerated no matter how complex the code.

The relationship between the average error and the transmission rate depends of course on the loss function that is used to measure the error. For any loss function we define the *rate distortion function* as the infimum of the transmission rate  $S_1$  of channels that can be used, together with some block code, so as to achieve the indicated average loss. This average loss is called the *distortion  $D$*  and the rate distortion function is written  $R(D)$ .

This is the model of the agent's information transmission problem that we use below. The principal gives the agent a channel with transmission speed  $S_1$ . The agent and subordinate arrange to use this channel in the most effective manner possible.

Let us look further into the nature of the decisions that the agent and subordinate can achieve in this way. Suppose, as much of information theory does, that the loss associated with each  $x_t$  depends only on the associated  $y_t$ . In our example with orders arriving at random, we would be assuming that a mistake in filling the order  $x_t$  engenders a loss for the agent if  $y_t \neq x_t$ . The magnitude of the loss depends on the nature of  $x_t$  and  $y_t$ . But any other  $y_{t'}$  is irrelevant to this loss. For instance, if  $x_t$  represents an order for 5,000 units in New York, and  $y_t$  is "send 4,000 units to Topeka," there is a loss. And if  $y_{t'}$  ( $t' > t$ ) is "send 3,000 units to New York," that does not mitigate the loss incurred by the mistake already incurred at  $t$ . In information theory such loss functions are called *single-letter fidelity criteria*.

Suppose a particular value of  $x_i$  is received. What will be the magnitude of the loss conditional on this event? After encoding, transmission, and decoding, there will be a value of  $y_i$ . But the  $y_i$  that results from  $x_i$  depends on the code word, which will typically be very long, in which  $x_i$  is imbedded. Different occurrences of the same realization of  $x_i$  will often be decoded differently, when a complex code is used. Therefore, the optimal code induces a transition probability  $q(y_i | x_i)$  which is not degenerate. For expected loss computations, it is this transition probability and the process  $x_i$  that are relevant.

The main result of information theory cited above can be reinterpreted in terms of the realized transition probabilities  $q(y_i | x_i)$ . A transition probability  $q(\cdot | \cdot)$  is attainable by an information system (a block code and a channel with a given transmission rate) if and only if the *average mutual information* of  $y_i$  and  $x_i$ ,  $I(y, x)$ , is less than this transmission rate. The average mutual information is defined as

$$I(y, x) = \int_Y \int_X p(x) q(y | x) \log \frac{q(y | x)}{q(y)} dx dy,$$

where  $p(x)$  and  $q(y)$  are the marginal distributions of  $x$  and  $y$ ,  $X$  and  $Y$  being the range of  $x$  and  $y$ .

Therefore, neglecting the approximation involved in the need to use arbitrarily long block codes to achieve the transmission rate, we can look upon the agent's decision problem as the minimization of the expected value of the loss function  $w(x, y)$  by choosing a family of conditional distributions  $q(\cdot | x)$  such that the average mutual information of  $x$  and  $y$  is below the transmission rate of the channel. This is a convex programming problem, whose solution can be found by means of constrained optimization methods.

It is important to realize that the randomness in  $y_i$  given  $x_i$  is not a "bad" thing. One can, with simpler codes, make  $y_i$  nonstochastic given  $x_i$ . In doing so, however, the problem is that many  $x_i$  will generally be coded as the same  $y_i$  and the average loss will be higher. Random  $y_i$  are a necessary by-product of block codes. And block codes are the key method through which limited channels achieve their best performance. Thus, when we optimize over  $q(\cdot | y)$ , we are implicitly incorporating the choice of a good code into the agent's design problem. We do not have to construct the code explicitly; indeed, the construction of such codes is itself a highly complex numerical problem. Its existence is guaranteed by the results of information theory cited above.

Any finite-length code that would transform blocks of  $x_i$  into a finite

set of possible  $z_t$  would therefore generate only a discrete set of decoded  $y_t$ . However, as we consider longer and longer code words, with an optimal encoding procedure, the regenerated  $y_t$  converge to a continuous distribution. Thus, since we have already accepted this limiting operation when we take the constraint  $I(x, y) \leq R$ , it does not represent any new assumption to treat the transitions  $q(\cdot | \cdot)$  as a family of continuous distributions.

The principal knows the agent's preferences and therefore can compute conditional distributions  $q_R(\cdot | y_t)$  that the agent would choose if given a channel with transmission rate  $R$ . The principal can choose  $R$  and must pay for its cost of installation (and maintenance). For simplicity, we assume that this cost is linear in the transmission rate. Therefore, the principal's problem is

$$\max \int_Y \int_X u(x, y) q_R(y_t | x_t) p(x) dy dx - cR,$$

where the expectation is taken over  $(x, y)$  with the exogenous process  $x$  and with  $y$  distributed according to the agent's chosen transition rule  $q_R(\cdot | \cdot)$  as described above.

In order to have a tractable form of this problem, we need a stationary process  $x_t$  and a single-letter fidelity criterion  $w(y_t, x_t)$  for the agent such that the resulting  $q_R(\cdot | x_t)$  takes a sufficiently simple form.<sup>4</sup> Although the basic properties of  $R(D)$  hold for very general processes and fidelity criteria, the resulting  $q_R(\cdot | x_t)$  usually cannot be computed in closed form. This makes evaluation and optimization by the principal impossible to carry out analytically.

The only case in which  $q(\cdot | x_t)$  takes a simple form is where  $x_t$  is Gaussian and  $w(x_t, y_t)$  is quadratic. This is the model of the agent that we study below. For the principal, we assume that  $u(x_t, y_t) = -(y_t - ax_t)^2$ .

The use of a continuous input and a continuous output requires a little comment because the discussion above was specifically directed to the case of a discrete set of inputs, strings of which could be encoded into strings of code letters. When the input is continuous, any encoding into a discrete set of code letters necessitates some error. An infinite transmission sequence would be necessary to send even one number with perfect accuracy. Nevertheless, the concept of a rate distortion function and the relationship of this function to the mutual information of  $x$  and  $y$  is still valid. The only qualification is that it must be possible to define a partition of the values of  $x$  into a family of sets with the cardinality of the code letters such that the average distortion within each element of the parti-



tion is finite. For the case of  $x$  normally distributed and the squared error fidelity criterion, this qualification is clearly satisfied (see Gallager 1968, Section 9.6, pp. 470-5).

#### IV The solution to the principal-agent problem

We assume that the stationary source  $x$  is Gaussian with mean 0 and variance  $\sigma^2$ , and we denote its density by  $p(x)$ . The agent maximizes expected utility under the given channel constraint by solving

$$\begin{aligned} \max - \int_Y \int_X (y-x)^2 p(x) q(y|x) dx dy \quad \text{subject to} \\ \int_Y \int_X p(x) q(y|x) \log \frac{q(y|x)}{q(y)} dx dy \leq R. \end{aligned} \quad (1)$$

From Gallager (1968) we know that the value of the agent's program at optimum is

$$u_A^* = -\sigma^2 e^{-2R}. \quad (2)$$

Moreover, this value is achieved by choosing a message  $y$  that has a conditional distribution

$$q^*(y|x) \rightsquigarrow \mathfrak{N}(\rho x, \rho \sigma^2 e^{-2R}), \quad (3)$$

where

$$\rho = 1 - e^{-2R}. \quad (4)$$

The optimal distortion has the appearance of a contraction toward the origin and the addition of a Gaussian noise with variance  $\rho \sigma^2 e^{-2R}$ .

The principal has a different objective function

$$u_P = - \int_Y \int_X (y-ax)^2 p(x) q(y|x) dx dy - cR, \quad (5)$$

where the cost of the channel  $R$  is supposed to be  $cR$ . The principal knows that the agent is maximizing his own objective leading to equations (3) and (4). Therefore, he looks for the best channel capacity  $R$  by maximizing equation (5) subject to equations (3) and (4).

Substituting equations (3) and (4) into (5) we rewrite the optimization program:

$$\max_{R \geq 0} - [e^{-2R}(2a-1) + (a-1)^2] \sigma^2 - cR. \quad (6)$$

A straightforward solution to this problem is characterized by the following relations. If  $a \leq \frac{1}{2} + c/4\sigma^2$ , the optimal channel capacity is  $R^* = 0$ ; the utility level of the principal is then  $u_P^* = -a^2\sigma^2$  and the utility level of the agent is  $u_A^* = -\sigma^2$ . If  $a > \frac{1}{2} + c/4\sigma^2$ , then

$$R^* = \frac{1}{2} \log \frac{2(2a-1)\sigma^2}{c},$$

$$u_P^* = -(a-1)^2\sigma^2 + \frac{c}{2} \left( 1 + \log \frac{2(2a-1)\sigma^2}{c} \right),$$

and

$$u_A^* = -\frac{c}{2(2a-1)} - \sigma^2.$$

Note that if  $c = 0$ , the principal either provides a channel of capacity that leads the agent to choose a constant action  $x = 0$  (which is also the principal's best decision without any information) or provides an infinite channel to let the agent achieve his first best.

If there were no incentives problem, the principal could achieve

$$\begin{aligned} & \max_{R \geq 0, q(\cdot|\cdot)} - \int_Y \int_X (y-ax)^2 p(x) q(y|x) dx dy - cR \\ & \text{subject to } \int_Y \int_X p(x) q(y|x) \log \frac{q(y|x)}{q(y)} dx dy \leq R, \end{aligned} \tag{7}$$

which yields

$$q^{**}(y|x) \rightsquigarrow \mathfrak{N}(\rho ax, \rho a^2 \sigma^2 e^{-2R}); \tag{8}$$

$$p = 1 - e^{-2R}; \tag{9}$$

$$R^{**} = \begin{cases} \frac{1}{2} \log(2a^2\sigma^2/c) & \text{if } 2a^2\sigma^2 \geq c, \\ 0 & \text{if } 2a^2\sigma^2 < c; \end{cases} \tag{10}$$

$$u_P^{**} = \begin{cases} -\frac{c}{2} [1 + \log(2a^2\sigma^2/c)] & \text{if } 2a^2\sigma^2 \geq c, \\ -a^2\sigma^2 & \text{if } 2a^2\sigma^2 < c. \end{cases} \tag{11}$$

The additional loss suffered by the principal due to the presence of an agent whose objective function differs from his own is plotted in Figure 1.

Given a cost function for the communication technology, the loss due to incentives increases monotonically with the amount of information to be transmitted. This result can be contrasted with results obtained in Green and Laffont (1982), where the loss due to incentives was not monotonic in the amount of information to be transmitted.

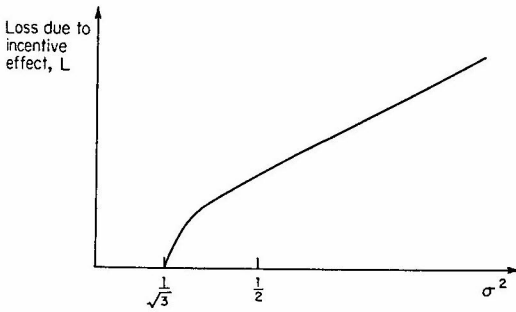


Figure 1.  $L = 1 + \log 3\sigma^2 - 3\sigma^2$  if  $\frac{1}{\sqrt{3}} < \sigma^2 \leq \frac{1}{2}$   
 $= \sigma^2 + \log \frac{3}{4}$  if  $\sigma^2 > \frac{1}{2}$

### V Social planning

A utilitarian social planner whose objective incorporated both the agent and principal and who could fully control the agent's behavior would solve

$$\max_{q(\cdot|\cdot), R} \int_Y \int_X [(y-ax)^2 + (y-x)^2] p(x) q(y|x) dx dy - cR \quad (12)$$

such that

$$\int_Y \int_X p(x) q(y|x) \log \frac{q(y|x)}{q(y)} dx dy \leq R.$$

Note that

$$(y-ax)^2 + (y-x)^2 = 2[y - \frac{1}{2}(a+1)x]^2 + \frac{1}{2}x^2(a-1)^2.$$

Using the change of variable,  $v = \frac{1}{2}(a+1)x$ , the problem becomes

$$\max_{q(\cdot|\cdot), R} -2 \int_Y \int_X (y-v)^2 p(v) q(y|v) dv dy - \frac{1}{2}(a-1)^2 \sigma^2 - cR \quad (13)$$

such that

$$\int_Y \int_X p(v) q(y|v) \log \frac{q(y|v)}{q(y)} dv dy \leq R,$$

where  $p(v)$  is the density function of  $v$ , that is, Gaussian with mean 0 and variance  $[\frac{1}{2}(a+1)]^2 \sigma^2$ .

From Gallager (1968) we know that the optimal solution with respect to  $q(\cdot | \cdot)$  is

$$q^*(y | v) \sim \mathfrak{N}(\rho v, \rho[\frac{1}{2}(a+1)]^2 \sigma^2 e^{-2R}), \tag{14}$$

$$\rho = 1 - e^{-2R}, \tag{15}$$

and that the value of  $\int_Y \int_X (y-v)^2 p(v) q(y | v) dv dy$  is

$$[\frac{1}{2}(a+1)]^2 \sigma^2 e^{-2R}. \tag{16}$$

The optimization reduces to

$$\min_R - \{ [\frac{1}{2}(a+1)]^2 \sigma^2 e^{-2R} - \frac{1}{2}(a-1)^2 \sigma^2 - cR \}, \tag{17}$$

yielding

$$R^{**} = \begin{cases} \frac{1}{2} \log[(a+1)^2 \sigma^2 / c] & \text{if } (a+1)^2 \sigma^2 \geq c, \\ 0 & \text{if } (a+1)^2 \sigma^2 < c; \end{cases} \tag{18}$$

and a social welfare

$$u^{**} = \begin{cases} -\frac{1}{2}(a-1)^2 \sigma^2 - \frac{c}{2}(1 + \log[(a+1)^2 \sigma^2 / c]) & \text{if } (a+1)^2 \sigma^2 \geq c, \\ -\frac{1}{2}(a-1)^2 \sigma^2 & \text{if } (a+1)^2 \sigma^2 < c. \end{cases} \tag{19}$$

Consider now what this social planner can achieve under the further constraint that the agent cannot be fully controlled, but rather that the agent will use the available channel to his own best advantage.

From Section IV we know that the agent chooses according to equations (3) and (4). The planner's objective function is then

$$-2a\sigma^2 e^{-2R} - [\frac{1}{2}(a-1)]^2 \sigma^2 - cR, \tag{20}$$

yielding

$$R^* = \begin{cases} \frac{1}{2} \log(4a\sigma^2 / c) & \text{if } 4a\sigma^2 \geq c, \\ 0 & \text{if } 4a\sigma^2 < c; \end{cases} \tag{21}$$

and a social welfare

$$u^* = \begin{cases} -\frac{1}{2}(a-1)^2 \sigma^2 - \frac{c}{2}[1 + \log(4a\sigma^2 / c)] & \text{if } 4a\sigma^2 \geq c, \\ -\frac{1}{2}(a-1)^2 \sigma^2 & \text{if } 4a\sigma^2 < c. \end{cases} \tag{22}$$

The additional social loss due to incentive problems is

$$L = \begin{cases} \frac{c}{2} \log[(a+1)^2 / 4a] & \text{if } \sigma^2 \geq c/4a, \\ \frac{c}{2} \log[(a+1)^2 \sigma^2 / c] & \text{if } c/(a+1)^2 < \sigma^2 < c/4a, \\ 0 & \text{if } \sigma^2 \leq c/(a+1)^2. \end{cases} \tag{23}$$

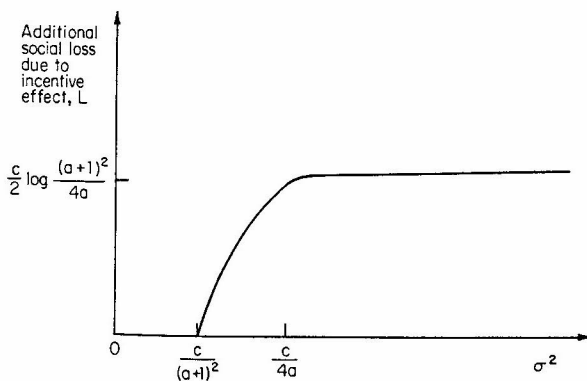


Figure 2

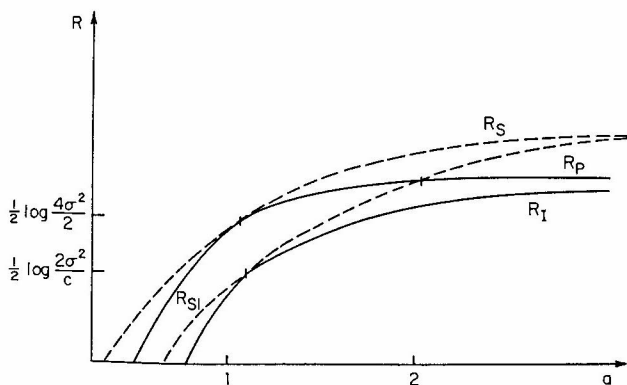


Figure 3

Here again the additional loss due to incentives is increasing monotonically in the information to be transmitted (see Figure 2).

We show in Figure 3 the optimal channel capacity or optimal expenditures on communication incurred in the various cases studied in this chapter:

- $R_P$  Principal-agent problem without incentives constraints
- $R_{P_I}$  Principal-agent problem with incentives constraints
- $R_S$  Social planner problem without incentives constraints
- $R_{S_I}$  Social planner problem with incentives constraints

Figure 3 is constructed under the assumption that  $c/\sigma^2 < 2$ . It is natural to focus on this case because in the opposite case the cost of the channel is so high that the principal would not like to use the channel even if the agent shared his objective function.

We observe the following results:

1. In both the principal-agent problem and the social planning problem, incentives constraints lead to a reduction in communication expenditures.
2. Under incentives constraints, expenditures in the principal-agent problem are always smaller than in the social planner's problem. Without incentives constraints this relationship continues to hold unless  $a$  becomes very large.

## NOTES

- 1 However, Marschak (1971) attempted to bridge economics and information theory.
- 2 In Green and Laffont (1982), we study this interaction when communication constraints are expressed as constraints on the dimensionality of the Euclidean spaces used by economic units to communicate their messages. See also Green (1982), Reichelstein (1983).
- 3 This theory was developed by Shannon (1948, 1959). Berger (1971) provides the most comprehensive treatment.
- 4 An algorithm due to Blahut (1972) could be used to solve these problems numerically.

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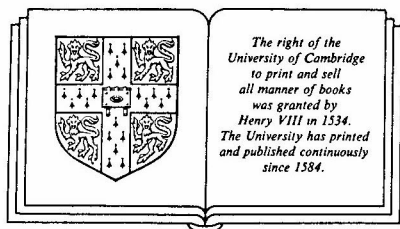
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