Commitments with Third Parties
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Commitments with Third Parties

Jerry R. GREEN *

ABSTRACT. – Observable irrevocable contracts between a principal and an agent have been suggested as a way in which the principal can enhance his payoff when playing a game against, or bargaining with, an opponent. It is shown that such beneficial agency relationships depend on the ability of the principal either to cut off further communication with his own agent or to cut the agent off from the opponent. With full communication, an unrestricted three-player bargaining phase will follow the contracting. In such a two-phase model, it is shown that the principal can never do better by employing an agent than he could have alone. Only contracts that result in either the principal's or the agent's payoffs being non-monotonic in the bargaining share they achieve have any potential to benefit the principal. But these contracts can be turned against him. The opponent can make another contract offer to the agent which, if superimposed on the principal's original contract, will result in an outcome that is actually worse for the principal than the solution of the original two-person problem.

Engagements avec des tierces parties

RÉSUMÉ. – Il a été suggéré que dans le cas où un principal et un agent sont engagés dans un jeu ou une négociation avec un opposant, le principal peut améliorer ses paiements par le recours à des contrats observables et irrévocables. On montre ici que de tels avantages dépendent de l'aptitude du principal à couper toute communication avec l'agent, ou à isoler l'agent de l'opposant. En cas de communication complète, une négociation à trois joueurs va suivre la phase contractuelle. Dans un tel modèle à deux phases, le principal ne peut jamais faire mieux en employant un agent qu'en agissant par lui-même. Il ne peut espérer tirer d'avantages que de contrats où ses paiements, ou ceux de l'agent, ne sont pas monotones dans leur part de la négociation. Mais ces contrats peuvent être retournés contre le principal. L'opposant peut en fait offrir à l'agent un autre contrat qui, superposé au contrat initial du principal, conduit à un résultat où ce dernier voit sa solution se détériorer par rapport à la solution du problème d'origine à deux personnes.

1 Introduction

A central idea in modern game theory, which has recently led to significant applications, was first stated systematically in Chapter 2 of Thomas Schelling's The Strategy of Conflict. Schelling points out that a player may benefit by diminishing his own payoffs, because this change influences the equilibria of the game he is playing. [1] Payoffs can be modified by contracts between the principal and a third party, the partner. Such a contract can be beneficial either because it alters the principal's payoffs when he continues to deal with his opponent or the market, or because it creates appropriate incentives for the partner when he takes over the principal's role and becomes an active strategic player.

These ideas rest on a two-phase conceptualization of the economic interaction. In the first phase, contracts are used to modify the payoffs of a game or a bargaining situation. In the second phase, the modified game is played, or the altered bargaining situation resolved.

Schelling recognizes that contracts made in the initial phase are not automatically credible. Arrangements with the partner must be observable and irrevocable if they are to be incorporated into the other player's understanding of the game and hence to influence the equilibria. Schelling writes:

> When one wishes to persuade someone that he would not pay him more than $ 16,000 for a house that is really worth $ 20,000 to him, what can he do to take advantage of the usually superior credibility of the truth over a false assertion? Answer: make it true... the buyer could make an irrevocable and enforceable bet with some third party, duly recorded and certified, according to which he would pay for the house no more than $ 16,000 or forfeit $ 5,000. [2]

But how is the contract made irrevocable? Secret or private renegotiation must be made impossible, or the contract will have no credibility. [3] To prevent renegotiation, one must either invoke a reputational argument or make communication between the principal and the partner impossible. [4] Finally, an effective contract requires that the partner not be able to contract with the opponent in a way that would undo the incentives

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1. Indeed Schelling shows this to be the case for payoff modifications that are, in every case, decreases, and hence easily credible when there is a form of free disposal of utility. This theme thas recently been explored by BEN-PORATH and DEKEL [1988].
2. SCHELLING [1960, p. 24].
3. See KATZ [1987] or DEWATRIPONT [1988].
4. With an explicit extensive form model of the contracting process, such as that in HART and MOORE [1987], there is a last moment at which legally binding commitments can be made; making a commitment at that time is a way of insuring that it cannot be modified. We shall not follow this route.

created for the partner's behavior in the game. As Schelling notes:

> In the example of the self-inflicted penalty through the bet, it remains possible for the seller to seek out the third party and offer a modest sum in consideration of the latter's releasing the buyer from the bet, threatening to sell the house for $ 16,000 if the release is not forthcoming. [5]

Assuming that the contract can be made observable, irrevocable, and immune to modification by further bilateral contracting between the partner and either the principal or his opponent, there remains the issue of how to model the resulting economic interaction. Who are the active players and what communication channels are open? If the principal can remove himself from the situation entirely, a two-person game is now to be played between the partner and the opponent. [6] If, as in Schelling's house purchase example, it is the partner who can be cut off from any remaining interaction, the principal's payoffs have been modified by virtue of his contract and it is he who still bargains with the seller, the partner playing no active role.

> The effect of the bet − as of most such contractual commitments − is to shift the locus and personnel of the negotiation in the hope that the third party will be less available for negotiation... If all interested parties can be brought into the negotiation the range of indeterminacy remains as it was... [7]

In this paper I shall re-examine this presumption. Could an observable irrevocable contract between a principal and his partner be beneficial when the parties know that the final result will be obtained through an interaction among all of them − that is, the principal can neither cut his partner off from the remaining bargain nor become unavailable himself. Is the "range of indeterminacy" the same as it was? That is, is the solution of the resulting three person bargaining problem the same, as far as the principal and his opponent are concerned, as the solution to their original two-person problem?

Schelling's analysis imposes one asymmetry between the principal and his opponent. The former has the first opportunity to make an offer to an outside agent to enter into a partnership with him, whereas the latter cannot contract with anyone. If he could, a four-player situation would result at

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5. SCHELLING [1960, p. 25].
6. This is the situation assumed in most of the recent purely theoretical literature; *see* KATZ [1987] and FERSHTMAN, JUDD and KALAI [1989]. Some of the applied literature presumes that the agent takes over the principal's possible moves in the game, while some assumes that the principal continues to act on his own behalf. *See* the further discussion below.
7. SCHELLING [1960, p. 25].

the renegotiation stage. We postpone the study of the symmetric four-person case to later work, concentrating in this paper on whether the unique opportunity to contract with a third player can be used to advantage. [8]

Although Shelling's ideas were well known and well appreciated, it is only very recently that the strategic advantages of contracting with outside agents have been examined in economic applications. For example, a firm can issue securities that effectively modify the return to the original equity holders much as if a Schelling-like "bet" had been made with an outsider. Here the third party is the debt market as a collective entity, rather than a single agent. The effect of such financial market strategies on product market behavior has been studied by BRANDER and LEWIS [1986] and MAXSIMOVIC [1986]. Other potential third parties are the firm's employees, especially if a union exists, as studied in DEWATRIPONT [1987, 1988], or even the firm's customers, if a long-term contract can be signed with them, as in AGHION and BOLTON [1987]. In this line of research the firm's original equity holders continue to be the strategic players in the product market interaction with actual and potential competitors. [9]

The other way of credibly altering product market behavior is to hire managers who are independent of the owners and who are compensated according to contractually determined performance measures. [10] Now the partner is the active player in the second-stage product market game, and the original equity holders are assumed able to withdraw credibly from further negotiation. This situation has been studied by SKLIVAS [1987] and FERSHTMAN and JUDD [1987].

Complications arise when there is incomplete information about firm characteristics that are relevant both to the third party and to the product market rivals. Signaling and contractual precommitment become intertwined. These issues have been studied by POITEVIN [1989 a, 1989 b, 1989 c] and GERTNER, GIBBONS and SCHARFSTEIN [1988].

Closer in objective to the present analysis are the papers of KATZ [1987], FERSHTMAN and JUDD [1986] and FERSHTMAN, JUDD and KALAI [1989]. They ask, in a general game theoretic context, whether and to what extent the original equilibria of the game between two principals can be altered when the principals are represented by agents whose payoffs are contractually determined.

These papers implicitly follow Schelling in assuming that the principals can isolate themselves from the actual playing of the game. With contracts in place, the game is played by the partner and the opponent, or by the two partners when each side has taken one. Here, however, I shall assume

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8. FERSHTMAN, JUDD and KALAI [1989], cited below, address this issue in the framework of non-cooperative game theory. This paper uses a cooperative approach, reflecting the unrestricted nature of the multi-person bargaining that follows the partnership and contract formation in our model.

9. These papers are directed at the issue of credible commitments between the equity holders and the third party. Therefoe their main focus is on making the contract secure against renegotiation.

10. *See* WILLIAMSON [1964, 1985].

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that the principals remain active participants. Their incentives may have been modified by the contractual arrangements they have made with their respective partners, but both they and their partners are players in the second-phase game.

Is this presumption of active principal behavior realistic? Consider some real-world agency relationships: lawyers under contract to negotiate between potential litigants, for example, or investment bankers retained by a raider and a target firm in a potential takeover, or the realtors acting for the buyer and the seller in a typical sale of a house. What is to stop one of the litigants from contacting the other and proposing a settlement without going through the attorneys? Why can't the target firm issue a public statement about some defensive actions it might take, effectively communicating with the raider without the investment banker's approval? Why can't the buyer negotiate specific terms directly with the seller of the house? Whether or not these contacts are advantageous, they do seem possible.

In these situations the actual negotiation is more complex than can be modeled by a two-person game involving only the two outside agents. Indeed the legal, ethical, and professional prohibitions regarding direct communication between a principal and the agent of the other party, or between the two principals once partners have been retained, testify that these activities are not irrelevant and that it takes severe sanctions to prevent their occurrence. [11]

These examples teach us a mixed lesson. Genuine isolated bilateral play between partners is probably not the right model of interaction, but neither is totally unrestricted three- or four-player bargaining. We shall nevertheless explore the latter possibility because previous research has concentrated on the former, and it is useful to establish another benchmark. I hope to discover whether Schelling's conclusions about the effectiveness of agency necessarily hinge on the conditions that renegotiation is impossible and only two of the three players can participate in the second phase of the interaction.

If the principal is able, for example, to cut himself off from all further communication, then the underlying situation is really not a symmetric bargaining model at all. In Schelling's example, if the buyer can offer his partner the incentive contract to buy the house and then become incommunicado, why can he not make a take-it-or-leave-it offer to the opponent, cutting off communication in just the same way? If he can cut himself off from the partner but not from the opponent, could not the opponent become an unwanted intermediary, carrying verifiable enforceable messages between partner and principal?

The paper proceeds as follows. Section 2 establishes the basic model, describing the nature of contracts and the three-person bargaining solution

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11. It would be interesting to study the creation and policing of the institutions through which such forbidden communication is monitored. This process probably involves the reputation of the agents, who are the long-run players, and the competitive environment in which they operate.

that is used to determine the outcomes. Section 3 shows that the ability of a principal to offer a contract that is beneficial to himself depends on the possibility of offering contracts whose results are not monotonically related to the underlying outcome of a bargaining game. Section 4 considers these beneficial nonmonotone contracts and asks whether there is some way that the opponent, even though he must move second, can counteract the adverse effect on his position by offering another contract. I show that if the original contract is required to depend only on the result of the underlying bargain, and not on the existence or nature of other contractual arrangements that the partner has entered into, then such countervailing contracts always exist and re-establish the symmetry of the simple underlying problem of pure division, as Schelling's intuition suggests.

However, if the principal can offer the partner a contract that enforces a punishment on the partner if he enters into any agreement with the opponent, then the principal can gain the upper hand. In this case the opponent, if he can counteract this stratagem at all, must do so by employing a partner of his own. This raises the rather difficult issue, pointed out by KATZ [1987], that each of the bilateral contracts may contain contingent clauses dependent on the contract that the other pair of parties has in place. We defer an analysis of this important matter to further work.

2 Contracts, Linear Solutions and the Iterative Linear Solution

In this section we consider a situation of pure bargaining between two players one of whom can contract with a partner. The players in the bargaining situation will be referred to as the principal, or player 1, and the opponent, or player 2. The partner will alternatively be referred to as player 3. Contracts between player 1 and his partner, player 3, are formalized as follows. The matter at issue is the division of a dollar between players 1 and 2. Let $t \in [0, 1]$ be the share of player 1 and $1 - t$ be the share of player 2. We will refer to t (or to the pair $(t, 1 - t)$) as the underlying allocation. A contract between player 1 and player 3 is a function $x : [0, 1] \to R$ with the following interpretation. If the underlying allocation is t, the contract gives $x(t)$ to player 3, taken out of player 1's share. Thus the resulting payoffs are $(t - x(t), 1 - t, x(t))$. Note that we do not require $x(t)$ to be non-negative. For example, if the partner gives the principal .25 as a lump-sum payment and then receives one half of the principal's share, $x(t) = -.25 + .5\, t$.

Once a contract is in place it is legally binding. That does not mean that it cannot be renegotiated. Nor does it mean that player 2 cannot counteract it by offering a bribe to player 3 not to invoke provisions of the contract. What it does mean is that if no further contracts are made, the

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original two person bargaining problem has been converted into a three person problem. Specifically, the underlying allocations of the original problem have been mapped into their image in $\mathbb{R}^3$ which we shall the contractually specified outcomes. In the example above, (z_1, z_2, z_3) is a contractually specified outcome if for some $t \in [0, 1]$, $z_1 = .25 + .5\ t$, $z_2 = 1 - t$, $z_3 = -.25 + .5\ t$. Since the renegotiation phase involves all three players it is natural to allow them to agree to a randomized underlying allocation instead of a deterministic value of t. Hence the feasible set of utility allocations is the convex hull of the set of contractually specified outcomes, which we will call the contractually specified feasible set, or bargaining region. The bargaining region is denoted $B(x)$.

$$B(x) = \text{co}\left\{ (z_1, z_2, z_3) \mid z_1 = t - x(t),\ z_2 = 1 - t,\ z_3 = x(t),\ t \in [0, 1] \right\}$$

To evaluate the efficacy of a contract the players must know what the resulting allocation will be. We will assume that the three person bargaining situation described by $B(x)$ is resolved by applying the iterative linear solution f^* which is described in detail below. [12]

The iterative linear solution is the limit of the following procedure. Let us define the linear solution F as a mapping from the set of all convex compact sets of R^3, B, into itself as follows. Take a vector of weights $u = (u_1, u_2, u_3)$ not all equal. Evaluate each outcome $z \in B$ by forming the inner product $u.z$ and find the maximizers over B, $m_u B$. Denote the permutations of the components of u by πu, where π is a permutation matrix. The linear solution generated by u is given by

(1) $$f_u(B) = (1/6) \Sigma_\pi m_{\pi u}(B)$$

The intuitive idea of a linear solution is that to select among points of B, in all of which the sum of individual's utilities are equal, one needs to have a social welfare function of some kind. Employing a linear social welfare function embodies the idea that if we were to solve two separate non-interacting problems independently one should get the same result as if we solved a single problem which is generated by considering them toge-

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12. An axiomatic derivation and analysis of this and related solutions is given in GREEN [1983]. Non-zero sum outcomes were studied by mapping them into equilavent zero sum bargaining situations to which the iterative linear solution, defined over zero-sum bargaining problems was applied. It is only these zero-sum problems that will concern us in the present paper.

  — The iterative linear solution is used in preference to some of the well-known cooperative bargaining solutions, such as the NASH [1950, 1953] solution or the KALAI and SMORODINSKY [1975] solution because these are unresponsive to the nature of the set of contractually specified outcomes. For example, the Nash solution would be invariant to this set whenever the solution occurs on the interior of $B(x)$. The Kalai-Smorodinsky solution, which depends only on the coordinatewise maximal utilities that could be obtained would be similarly unresponsive, and indeed might not even lie in $B(x)$. The iterative linear solution seems to have good properties in these respects, although since it has not been axiomatically characterized, it is not on as firm a footing as these solutions in this respect. The superadditive solution of MASCHLER and PERLES [1981], which is closest in spirit to the linear solutions of GREEN [1983], does not exist for three or more players. *See* PERLES [1982].

ther. [13]   A given set of relative weights applied to the problem will not, of course, treat the players equally.   To symmetrize the selection of an outcome based on a set of weights we imagine that the three relative weights are fixed and that they are assigned at random to each of the three players.   Thus, as expressed in equation (1), the six permutations of u generate six outcomes which will each be extreme points of B, and the linear solution generated by u will be the mean of these outcomes.

Thus it is natural to define the linear solution as a correspondence, $F: B \rightarrow R^3$ by

$$F(B) = co \left\{ z \in R^3 \,\middle|\, z \in f_u(B) \text{ for some } u \right\}$$

An equivalent way to think about the outcomes identified by F is to imagine the players trying to justify outcomes $z \in B$ to an outside mediator.   Each $z \in B$ can be justified if there exists a vector of weights u such that $z \in f_u(B)$. [14]   The idea is that if $z \notin f_u(B)$ for any u, then no player could argue before the mediator that z should be the outcome.   He could not say that had weights u been used to generate an anonymous procedure, that anonymous procedure would have selected z.

If one accepts the linear solution as a way of narrowing down the outcomes to a subset of B, I would argue that it is logical to continue the process and iterate the linear solution by applying it to F(B).   The original disagreement of the players about which point in B to choose can be recast as a disagreement about which weights, or which randomized weights, to use as a generator of the linear anonymous social welfare function.   This dispute should be settled no differently than if this set of utility combinations had arisen directly as a contractually specified feasible set rather than indirectly as the image of some such feasible set under the linear solution.   We denote the limit of the iterates of F applied to a contractually specified feasible set B by $f^*(B)$.

In the remainder of this paper we will assume that the players evaluate contracts x by considering $f^*(B(x))$.   Player 3, in particular, is willing to accept the offer of x if and only if $f_3^*(B(x)) \geqslant 0$.   This reflects the partner's opportunity cost.   The principal's problem is to maximize $f_1^*(B(x))$ over all contracts x such that $f_3^*(B(x)) \geqslant 0$.   Because the solution used to model the negotiation phase is translation invariant, this problem is obviously equivalent to choosing the x that minimizes $f_2^*(B(x))$, since the principal could extract any surplus the partner earns by adding or subtracting a constant from x for all values of t.

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13. *See* MASCHLER and PERLES [1981].   A further discussion and additional arguements for the additivity (or super-additivity) of solutions is given in MOULIN [1988].
14. In GREEN [1983] where continuity is imposed, the requirement is that z should remain in $f_v(B)$ for all v in a neighborhood of u.   This results in continuity for each set of smoothly randomized weights but the solution F(B) defined by (2) is not a closed set in all cases.   Thus iterating the mapping F will not always be well-defined.   In order to achieve a single-valued solution, which is necessary if the players are to evaluate the results of contracts, we have chosen to give up the continuity requirement and work directly with $f_u(B)$ for each u.

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If there exists a contract x^* such that $f_2^*(B(x)) < 1/2$, we will say that the principal can benefit from contracting (with a partner). If not, then contracting is not beneficial because the principal can achieve the payoff $1/2$ by remaining in the original two-player bargaining model. [15]

We conclude this section with a brief description of the workings of the iterative linear solution, as this is central to the results in the remainder of the paper. Observe first of all that we can restrict attention to $u \in R^3$ which have norm one. All linear solutions depend only on the relative weights expressed by u and not on the scale of u. To insure that the components of u are not all equal we can impose the condition that they sum to zero. Thus we take $u \in S$ where,

$$S = \{ u \in R^3 \mid \| u \| = 1 \text{ and } \Sigma u_i = 0 \}$$
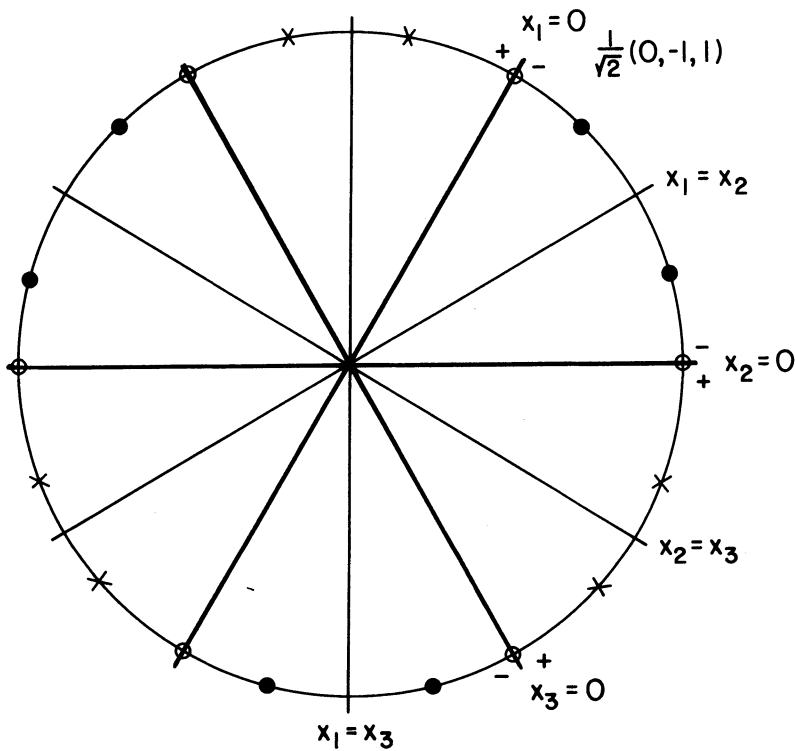
A picture of S is shown in Figure 1.

FIGURE 1

The circle is the set $S = \{ u \in R^3 \mid \| u \| = 1, \Sigma u_i = 0 \}$.
The straight lines show the intersection of $\{ u \in R^3 \mid \Sigma u_i = 0 \}$ with various planes whose equations label these lines.

15. All standard solution concepts for two person bargaining games coincide in zero-sum cases and yield the midpoint of the relevant range. Moreover, the two person version of the iterative linear solution with which we are working leads to the same result; *see* footnote 11 below.
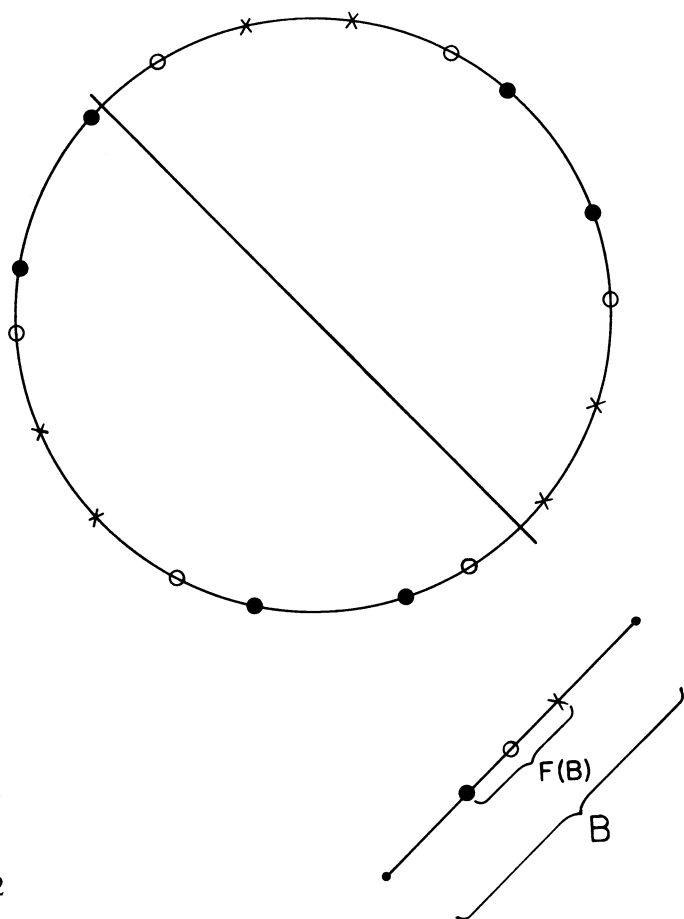
FIGURE 2

The Linear Solution Applied to a Segment B in $\mathbb{R}^3$.

Take any $u \in S$. We are particularly interested in the values of πu for the six permutations π. Three of these u are shown in Figure 1. The points denoted by the open circle are the permutations of $u = (1/\sqrt{2})(1, 0, -1)$; those denoted by the solid dot are the permutations of $u = (\gamma, -\delta, -\delta)$, for γ, $\delta > 0$; and those denoted by the cross are the permutations of $u = (-\gamma, \delta, \delta)$ for γ, $\delta > 0$.

Figure 2 shows how the linear solution applies to the simplest convex set in $\mathbb{R}^3$, the line segment connecting two points, denoted B. When we take $u = (1/\sqrt{2})(1, 0, -1)$, there are three values of πu that lead to $m_{\pi u}(B)$ at either endpoint of the interval. However, the other two values of u shown in Figure 1 each lead to two of the six $m_{\pi u}(B)$ at one endpoint and the remaining four at the other. Thus the linear solution $F(B)$ is the middle third of the interval. Obviously, the iterative linear solution $f^*(B)$ is the midpoint of the interval. [16]

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16. In a two dimensional bargaining problem the iterative linear solution is also the midpoint of an interval, but no iteration is required. There is only one permutation invariant measure on $\{ u \in \mathbb{R}^2 \, | \, u_1 + u_2 = 0 \text{ and } \|u\| = 1 \}$, one half the weight on each endpoint, and this immediately results in the midpoint being selected, without any iteration required.

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Note that the null contract $x \equiv 0$ results in the contractually specified feasible set being one side of the simplex Δ^3, that is, a line segment. Thus, we can state:

PROPOSITION 1: When either the null contract is offered, or if no contract is offered, the iterative linear solution results in $f_2^*(B(x)) = 1/2$.

Figure 3 depicts the application of the linear solution to a more general set B. In this case

$$B = \text{co} \left\{ (0,\ 1,0),\ (.7,\ .3,\ 0),\ (0,\ .4,\ .6),\ (.55,\ 0,\ .45) \right\}$$

and

$$F(B) \approx \text{co} \left\{ (.23,\ .57,\ .20),\ (.30,\ .45,\ .25),\ (.18,\ .47,\ .35),\ (.42,\ .23,\ .35) \right\}$$

Note that $F(B)$, although a quadrilateral, is not similar to B. For more general sets B the shape of $F(B)$ can differ quite markedly from that of B.
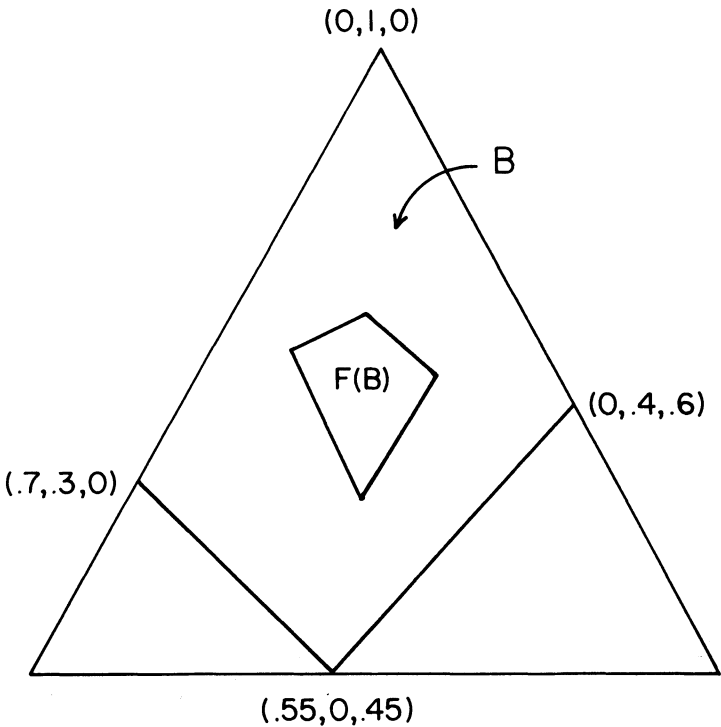

FIGURE 3

The Linear Solution Applied to a More General B.

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3 Monotone and Non-Monotone Contracts

In this section we ask whether there are advantageous acceptable contracts that can be offered by the principal. The results are very simple. Let us define a contract to be monotone if the principal's payoff $t - x(t)$ and the partners payoff $x(t)$ are both weakly monotonic increasing in t. We will show

THEOREM 2: Given any monotonic contract x, $f_2^* = 1/2$.

Thus by Proposition 1, all monotone contracts have the same payoff as the contract $x \equiv 0$.

Conversely we will demonstrate

THEOREM 3: For any $\varepsilon > 0$, there exist non-monotonic contracts x such that $f_2^*(x) < \varepsilon$.

We begin by developing some simple intuition about the behavior of f^* on the bargaining regions that can be achieved by various contracts.

Let us say that x is a simple incentive contract if

$$\text{(2)} \qquad \begin{cases} x(t) = 0, & t \leqslant t_1 \\ x(t) = t - t_1, & t > t_1 \end{cases}$$

A simple incentive contract is shown in Figure 4.

In a simple incentive contract the partner receives all the bargaining surplus beyond a fixed level but nothing if the principal's share does not reach that level. In an actual bilateral negotiation between the partner and the opponent, one would imagine that the partner would put in a lot of effort or would take a great deal of downside risk in order to increase the probability of the result exceeding t_1. Of course the idea of simple incentive contracts is that by encouraging the partner to try for this upper tail the principal will benefit since he receives t_1 whenever the partner receives anything at all.

A basic result, central to all that follows, can now be stated:

PROPOSITION 4: For all simple incentive contracts x, $f_2^*(x) = 1/2$.

Proof: Consider a simple incentive contract as specified in (2) which generates a bargaining region B. Let us compute the linear solution applied to B. We will show that $F(B)$ is a triangle similar to B and has as one of its sides the middle third of one of the sides of B. Denote the vertices of B as follows:

$$a = (0, 1, 0)$$
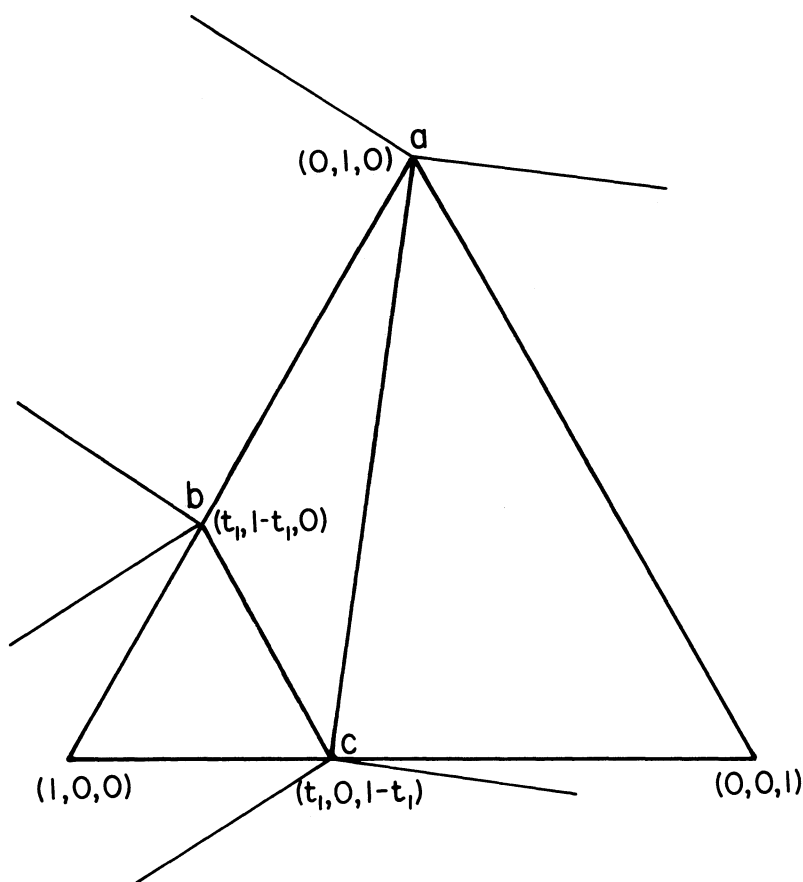$$b = (t_1, 1 - t_1, 0)$$
$$c = (t_1, 0, 1 - t_1)$$

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FIGURE 4

A Simple Incentive Contract.

We will first show that vertices a and c must each receive weight $1/3$ for any $u \in S$; that is, given $u \in S$ there are at least two of the πu such that $m_{\pi u}(B) = a$ and at least two such that $m_{\pi u}(B) = c$. Take the case where $u_1 < u_2 < u_3$ — cases where some components of u are equal will be treated separately. The permutations $\pi_1 u = (u_1, u_3, u_2)$ and $\pi_2 u = (u_2, u_3, u_1)$ will both result in $m_{\pi u}(B) = a$. Likewise, $\pi_3 u = (u_2, u_1, u_3)$ and $\pi_4 u = (u_1, u_2, u_3)$ will both induce $m_{\pi u}(B) = c$.

Now if $u_1 = u_2 < u_3$ the above argument still applies. But if $u_1 < u_2 = u_3$, then $\pi_3 u$ still induces $m_{\pi u}(B) = c$ but $\pi_4 u$ now induces $m_{\pi u}(B) = [b, c]$ and not c alone. However $\pi_5 u = (u_3, u_1, u_2)$ induces $m_{\pi u}(B) = c$, so there are at least two values of πu inducing c in all cases.

Next we observe that when $u_1 < u_2 = u_3$, $m_{\pi u}(B) \neq b$ for any π. Clearly $\pi u.b$ is maximal when $\pi u = (u_2, u_3, u_1)$, but in this case $\pi u.a = \pi u.b = u_2$, so $m_{\pi u}(B) = [a, b] \neq b$. For $\pi u = (u_1, u_2, u_3)$, $\pi u.b < \pi u.a$. And for $\pi u = (u_2, u_1, u_3)$, $\pi u.b < \pi u.c$. Thus at this u, $m_{\pi u}(B)$ can be selected on the segment ac for all π.

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Finally, when $u_1 > u_2 = u_3$, $m_u(B) = [b, c]$, and by selecting b, $f_u(B) = 1/3(a+b+c)$. Thus $F(B)$ is the triangle with vertices $a' = 2/3\,a + 1/3\,c$, $b' = 1/3(a+b+c)$, $c' = 1/3\,a + 2/3\,c$.

Applying the linear solution to $F(B)$ we see that $f^*(b) = 1/2(a+c)$ and thus $f_2^*(B) = 1/2$. $\square$

Thus no simple incentive contract can be beneficial to the principal, despite the fact that these contracts offer the partner the complete marginal incentive to bargain for a payoff exceeding the level t_1. Indeed, as the next result shows, the issue of bargaining for higher outcomes is not really relevant at all because of the cooperative nature of the solution used to resolve the bargaining problem among all three players. We shall say that x is an inverted incentive contract if

$$x(t) = t \qquad t \leqslant t_1$$
$$x(t) = t_1 \qquad t > t_1$$

While inverted incentive contracts seemingly do not give the partner the incentive to bargain for a payoff that would be at all beneficial to the principal, the following proposition shows that they are no worse than, and equally as ineffective as, simple incentive contracts. [17] The result is presented only because it sets the stage for the analysis of more general monotone contracts.

PROPOSITION 5: For all inverted incentive contracts x, $f_2^* = 1/2$.

The method of proof is the same as that for Proposition 4 and is omitted. The value of the solution is $f_1^* = 1/2(1 - t_1)$, $f_2^* = 1/2$, $f_3^* = 1/2\,t_1$.

Now let us consider a general monotone contract x such as that shown in Figure 5 and its associated bargaining region B. Let $(\alpha, 0, 1-\alpha)$ be the allocation determined by the contract at $t = 1$. Let P be the parallelogram with vertices at $(0, 1, 0)$, $(\alpha, 1-\alpha, 0)$, $(\alpha, 0, 1-\alpha)$. By virtue of the monotonicity of x, $B \subseteq P$.

Observe that the middle third of the diagonal of P connecting $(0, 1, 0)$ and $(\alpha, 0, 1-\alpha)$ is inclued in $f(B)$ and that no points on this diagonal are in $f(B)$, by precisely the same reasoning as that used in Proposition 4. Let P_1 be the parallelogram similar to P having as its diagonal the middle third segment of the diagonal of P. Thus P_1 is formed by the constraints

(3) $\qquad\qquad x_1 \geqslant \alpha/3$

(4) $\qquad\qquad x_1 \leqslant 2\alpha/3$

(5) $\qquad\qquad x_3 \geqslant (1-\alpha)/3$

(6) $\qquad\qquad x_3 \leqslant 2(1-\alpha)/3$

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17. Here, as in the bet suggested by Shelling in *The Strategy of Conflict*, if the principal and the opponent can bargain they will each get $(1 - t_1)/2$. But the principal can extract t_1 from the partner and so obtain a total payoff of $(1/2) + t_1/2$. Thus the optimal inverted incentive contract is approximated by setting $t_1 = 1$, a contract in which the partner receives all the gains from bargaining with the opponent, and is thus interpreted as the active player in the partnership with the principal.
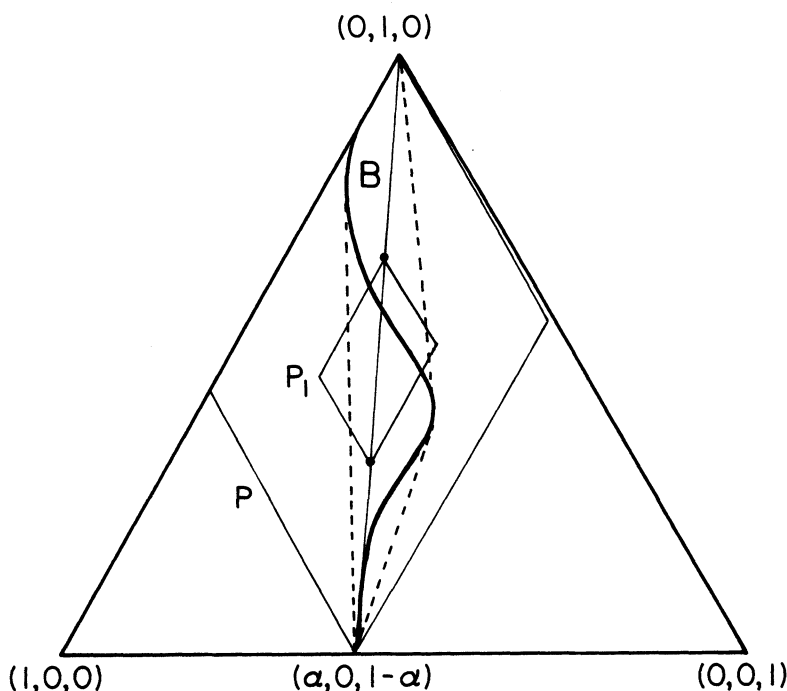
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$$(0, 1, 0)$$

B

P_1

P

$$(1, 0, 0) \qquad (\alpha, 0, 1-\alpha) \qquad (0, 0, 1)$$

FIGURE 5

A Simple Monotonic Contract.

We will show that $f(B) \subseteq P_1$ and thus that the process can be iterated to obtain convergence to $f^*(B) = (\alpha/2, 1/2, (1-\alpha)/2)$.

Take $y \in f(B)$; we want to show $y \in P_1$. We know that y is a linear combination of the extreme points of B and that $(0, 1, 0)$ and $(\alpha, 0, 1-\alpha)$ receive weight of least $1/3$ each, leaving at most weight $1/3$ for the remaining extreme points. Express each of these extreme points as a positive linear combination of vertices of P, and rewrite y as a linear combination of these four points as follows:

$$y = \beta_1 (0, 1, 0) + \beta_2 (\alpha, 0, 1-\alpha) + \beta_3 (\alpha, 1-\alpha, 0) + \beta_4 (0, \alpha, 1-\alpha)$$

We know that $\beta_1 \geqslant 1/3$, $\beta_2 \geqslant 1/3$.

Now we can verify directly that y must satisfy that inequalities (3)-(6) that define P_1. From $\beta_2 \geqslant 1/3$, we have (3) and (5) directly. If $y_1 > 2\alpha/3$, then $\beta_2 + \beta_3 > 2/3$; but this contradicts $\beta_1 \geqslant 1/3$. Likewise, if $y_3 > 2(1-\alpha)/3$, then $\beta_2 + \beta_4 > 2/3$, again contradicting $\beta_1 \geqslant 1/3$.

Summarizing the above argument gives us Theorem 2.

Theorem 3 is demonstrated constructively, as shown in Figure 6. The principal can offer the partner a contract in which the latter receives more than all the marginal gain from increases in t, beyond some t_1. This results in a set $B(x)$ as shown. When the linear solution is applied to such sets,

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the point in B(x) at which $t = 0$ receives less weight than in the case of monotone solutions. In recursive applications, the iterates of F are closer to the side of B along which the partner is experiencing this more than full marginal gain. As these points are associated with low payoffs for the opponent, the value of f_2^* can be made arbitrarily small. A special case of interest that does not require the principal to gain at the partner's expense for low values of t is the contract where the partner gives up a lump-sum payment close to 1/3 and receives nothing unless the value of t is very close to unity, in which case the partner gets all of t and the principal receives nothing. The limit of such contracts produces the entire simplex (translated by 1/3 from the partner to the principal) as its bargaining region; and this results in the payoffs (2/3, 1/3, 0). [18]

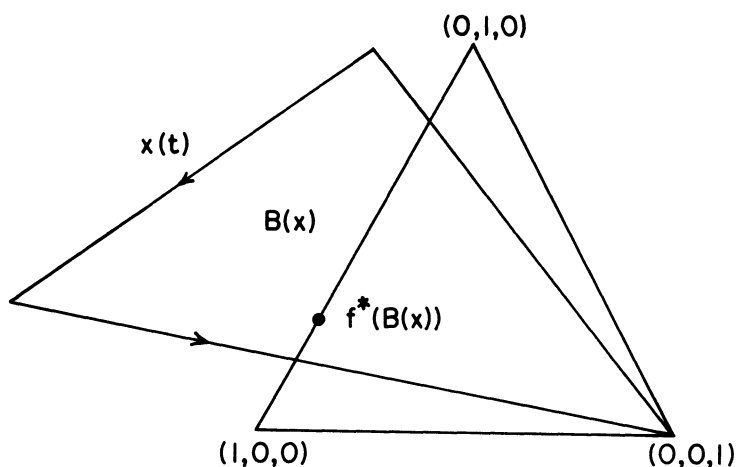

FIGURE 6

A Non-Monotonic Contract with $f_2^*(B(x)) < 1/2$.

4 Countervailing Contracts

Because of the results of Section 3, we know that a contract that is beneficial to the principal will be non-monotonic. Suppose that x is such a contract and that $f_3^*(x) = 0$, without loss of generality. To counteract the effects of x the opponent may contract with the partner. The original contract between the principal and the partner remains in force. But

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18. Kathy Spier pointed this contract out to me.

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superimposed upon it is a new agreement $y(t)$ representing the transfer from the opponent to the partner, as a function of the underlying allocation. With both contracts in place the resulting payoffs are:

$$z_1(t) = t - x(t)$$
$$z_2(t) = 1 - t - y(t)$$
$$z_3(t) = x(t) + y(t).$$

Let $B_x(y)$ be the resulting bargaining region where the contract y is superimposed upon the result of x. We will say that y is a countervailing contract against x if

$$f_3^*(B_x(y)) \geq 0$$
$$f_2^*(B_x(y)) \geq 1/2.$$

The main result of this section is that there is always a countervailing contract. In core-like terminology, not only can any contract x that is acceptable to the partner and gives the principal more than $1/2$ be blocked, it can be blocked by a contract y that gives the principal less than if he had never offered x at all and gives the opponent more than if the principal had not offered x. [19] Interpreting contracts in our model as cooperative allocations in a characteristic function game, we see that the core as usually defined is empty. Our main result can also be cast in terms of the "bargaining set". Since there are no beneficial contracts between the principal and the partner which cannot themselves be "blocked" by countervailing contracts, the "bargaining set" is simply the original "no contract" situation in which the principal and the opponent are engaged in a game of pure division.

Let us now turn to a demonstration of this result.

LEMMA 6: Given any non-monotonic contract x, there exists a contract y such that $f_1^*(B_x(y))$ equals the minimum payoff to the principal, $t - x(t)$, at any contractually specified outcome.

We sketch a constructive proof of Lemma 6, without giving all the details, as follows. The bargaining region associated with the original contract can be expanded by the countervailing contract in a way that mandates large transfers from the partner to the opponent for some values of t, large transfers in the opposite direction for other values of t, and no transfer in other cases. As shown in Figure 7, we can make these large positive and negative transfers in cases where the principal's payoff in the contractually specified outcome is low, and have no transfer between the opponent and the partner for values of t where the principal's payoff is high. This construction results in a new bargaining region $B_x(y)$ in which the iterated linear solution selects a point that can be made as close a possible to the boundary of the bargaining region in which the principal's payoff is at its minimum, as stated in the lemma.

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19. Note that all allocations are efficient in this pure bargaining setting. Hence, this "blocking" does not involve destructive activity as is often the case in analyses of the core.
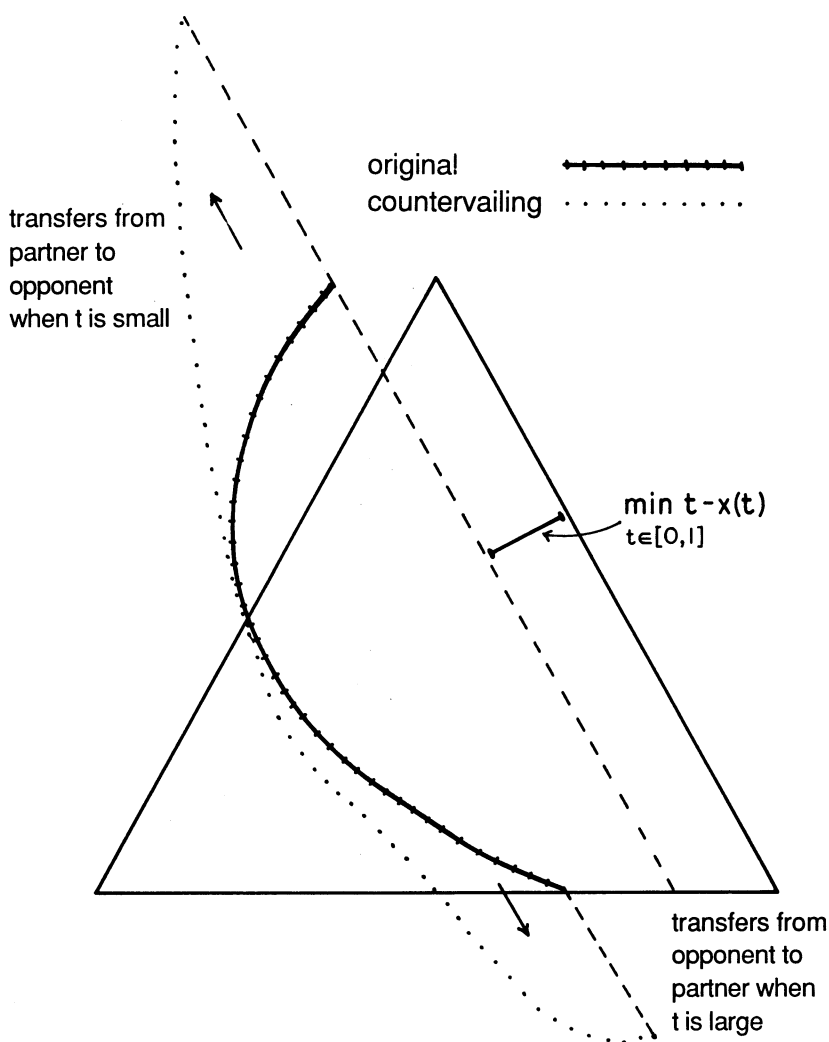
original ·—·—·—·—·—·
countervailing · · · · · · · · · ·

transfers from
partner to
opponent
when t is small

min t−x(t)
t∈[0,1]

transfers from
opponent to
partner when
t is large

FIGURE 7

LEMMA 7: If x is a contract such that $f_3^*(x) \geqslant 0$, then there exists some t for which $t - x(t) \leqslant 1/2$.

Again, only a sketch of the proof is given; the argument is straightforward. If in the original contract x, contrary to hypothesis, the principal's payoff was uniformly greater than 1/2, the bargaining region would be bounded below and to the left of the line AB in Figure 8. Observe first that the iterated linear solution for the bargaining region given by the interval AB is the allocation (1/2, 1/2, 0). Any other bargaining region subject to this constraint can be generated by a contract that is at least as favorable, for all values of t, as the contract generating AB. Therefore the result of the iterated linear solution is necessarily at least as favorable for the principal, and cannot be any more favorable for the partner. Therefore such contracts will fail to satisfy the constraint $f_3^*(x) \geqslant 0$.
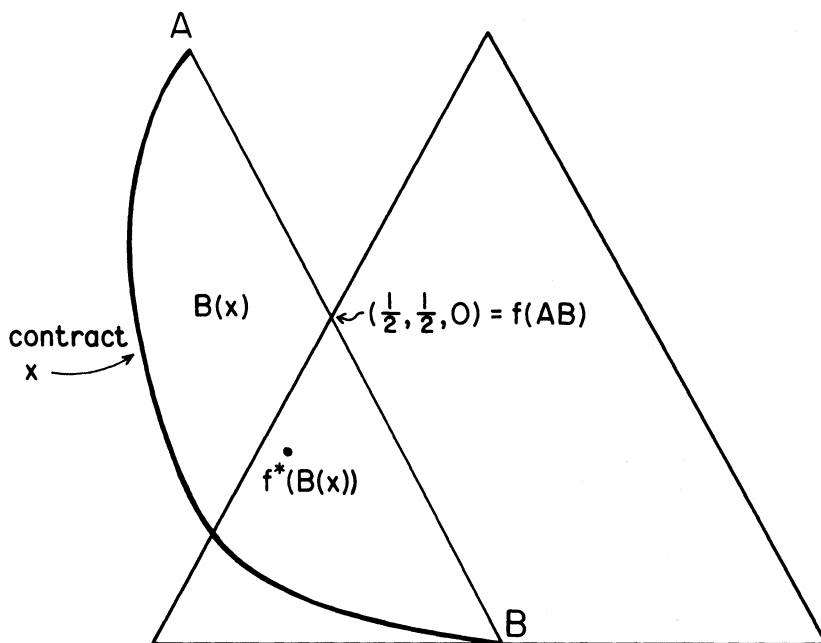
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FIGURE 8

Combining these results we see that any initially advantageous contract can be modified by a countervailing contract in which the principal fairs worse than if he had not offered any contract to a partner at all. This is summarized as:

THEOREM 8: If x is a contract that is acceptable to the partner and beneficial to the principal, then there exists a contract y that is a counter-vailing contract against x.

5 Conclusion

We have demonstrated that even though only the principal can form a contractual relationship with an outside agent for the purpose of influencing the results of a bargain against his opponent, there is reason to believe that such a relationship will not be advantageous. If the contract is monotone, then it will not have any effect. And if it is non-monotone, it can be effectively thwarted by a countervailing contract formulated by the opponent. As Schelling has suggested, the possibility of unrestricted negoti-ation of the outcome after observable contracts have been put in place removes their strategic value. The solution of this bargaining problem is

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the same as if there were no contract at all and the principal faced the agent directly.

In order for there to be any advantage for the player who has the first opportunity to offer a contract to the agent, this contract must include provisions that stop the partner from subsequent contracting with the opponent. The existence of contracts such as these might force the opponent to engage a partner of his own. Then the resulting unrestricted bargaining would involve four players rather than three.

Though I suspect that the principal result of this paper would survive this generalization, one cannot be certain. A serious theoretical difficulty exists, as pointed out by KATZ [1987]. When both bilateral contracts can contain stipulations regarding each other, and in principal stipulations concerning the stipulations made in the other, ad infinitum, the actual environment in which the second stage bargaining would take place may not be well-specified. We hope to address this issue in further work.

The principal methodological conclusion to be drawn from the results of this paper is that there is reason to be cautious in using models where one of the competing principals gains an advantage merely from using a partner in a game that he could equally well play himself. Since agency relationships are so common, both in games and in bargaining, the reason must lie elsewhere. Either the partner has superior information or abilities, or the partner has a strategic advantage in playing the game, such as a long-term reputation to protect. Models that ascribe agency to purely strategic considerations are able to do so only because of their explicit or implicit assumption that communication channels used for multilateral bargaining can be cut off.

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