The Weighting Game: Formula Apportionment as an Instrument of Public Policy

Abstract - *We propose an explanation for why states choose different apportionment formulas for corporate income tax purposes. Based on a two-state equilibrium model of location choice by firms, we show that aggregate social welfare is maximized when both states use the same formula, regardless of which formula is chosen. However, at least one of the states can increase its welfare by deviating from this coordinated solution; thus, the Nash equilibrium features the states choosing different formulas. Importing states have incentives to increase the sales factor, whereas exporting states will tend to increase the input factors. An empirical test of which states have deviated from the traditionally equally-weighted three factor formula supports the predictions of the model.*

INTRODUCTION

Forty-six of the 50 states in the United States impose a corporate income tax at the state level. If a corporation has business activities in multiple states, then each state with which the corporation has sufficient contact for the state to tax its income (nexus) can levy a tax on the income earned within that state. Measuring income earned within each of several political jurisdictions presents a difficult conceptual problem. To deal with this problem, the states have adopted a system of formula apportionment to allocate income among states. If each state adopts the same apportionment formula, then exactly 100 percent of any corporation's income will be apportioned among the 50 states for corporate income tax purposes. Lack of uniformity across states can result in either more or less than 100 percent of a corporation's income being subject to state income tax.

The Multistate Tax Compact was established in 1967, in an effort to increase uniformity of treatment across states. Article IV of the compact provides that business income is to be apportioned among the states on the basis of the corporation's portion of property, payroll, and sales in each state, and that the three factors are to be weighted equally. For example, if a corporation has 50 percent of its property, 40 percent of its payroll, and 15 percent of its sales in Alabama, then

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183

(50% + 40% + 15%)/3 = 35 percent of its income would be taxed by Alabama.

Prior to 1978, it was not clear whether deviating from the three factor, equal weighted formula (EWF) was constitutional under the commerce clause. The Supreme Court upheld the right of states to deviate in *Moorman Manufacturing Company v. Bair*, 437 U.S. 267 (1978). In the twenty years since *Moorman* was decided, about two-thirds of the states that impose a corporate income tax have deviated from EWF. The most common deviation from EWF is to double weight the sales factor (DWSF). Suppose the corporation in the example above had the remaining 50 percent of its property, 60 percent of its payroll, and 85 percent of its sales in Georgia, which uses DWSF. Then (50% + 60% + 2* (85%))/4 = 70 percent of its income would be taxed by Georgia. Lack of uniformity in formulas results in 105 percent of the corporation's income being subjected to state corporate income tax.

Proponents of using DWSF argue that it is an effective economic development tool. Compared to the EWF, the DWSF decreases the tax burden on firms producing within that state and exporting to another state, while increasing the tax burden on firms that produce in other states and import into that state. Ceteris paribus, a state using the DWSF will be a more attractive place to locate the property and payroll of a business enterprise operating in multiple states than a corresponding EWF state. In a critique of DWSF on constitutional grounds, Simafranca (1995) asserts that "[G]enerally speaking, every state has an economic incentive to increase the weight assigned to its sales factor" and "If a state were to double weight the property and payroll factors, it would experience an economically detrimental result."

This argument, however, is at odds with the facts. After more than 30 years since the Multistate Tax Compact (MTC), almost one-third of the states still equal-weight the three factors, suggesting that states need not have the same incentives. In addition, the incentive to attract production into a state would appear to be only one of many conflicting components of a state's welfare objectives. Thus, it would be natural to examine the incentives of states to alter formula weights under a more general framework.

Variation in incentives across states can only result if states differ in some underlying characteristics. In particular, with homogeneous preferences across states, and all mobile inputs, states should not have conflicting objectives. We argue that the choice between EWF and DWSF is explained by the desire of states to tax immobile capital, such as agriculture and natural resources, rather than the desire to attract mobile capital, such as manufacturing. Heterogeneity across states with respect to possession of natural resources creates a conflict of interest among states, inducing states that export (import) the output from immobile capital to put less (more) weight on the sales factor.

We analyze a setting in which states differ in their demand for and endowments of immobile inputs (e.g., land). Using a simple two-state equilibrium model of firm production choice based on these primitives, we examine the economic incentives of states to select different apportionment formulas.

In the second section, we describe the model and characterize the equilibrium. We then show that aggregate social welfare is maximized when the two states choose the same formula, regardless of which formula is chosen. However, if each state acts to maximize its own welfare—taken to be the sum of its tax revenues, producer surplus in its state, and consumer surplus in its state—then at least one and perhaps both states have an incentive to choose a formula different than any coordinated outcome. Therefore, the socially efficient outcome can never be attained if each state acts to maximize its

184

own welfare. This is consistent with the observed differences in state apportionment formulas, and suggests federal constraints on state formulas would increase social welfare.

In the third section, we conduct a simple empirical test to examine the extent to which states behave in the manner we suggest. We find the estimating model, though parsimonious, to have strong predictive power. In the fourth section, we discuss some implications of our findings, as regards both the existing wisdom on the logic underlying states' choices of apportionment formulas, and the outcomes if a formula apportionment system were to be implemented in the international context.

Our paper is related to a sparse, but growing, literature on formula apportionment. Gordon and Wilson (1986) examine the response of firms to a system of formula apportionment, restricting attention to cases in which all states use the same system, with different corporate tax rates. In contrast, we study the case in which the apportionment choices can vary across states. Goolsbee and Maydew (2000) find that states that have increased the weight on the sales factor have experienced faster employment growth than have EWF states. Finally, Weiner (1996) and Klassen and Shackelford (1998) document the association between state taxes and investment behavior and sales strategies, respectively.

MODEL

The Basic Setup

We study an economy with two political jurisdictions (states) and one tradable good. Demand for this good is perfectly inelastic but differs across states.[1] Thus, the quantity demanded is δ_x in state x, and is δ_y in state y. We assume that $\delta_y > \delta_x$, without loss of generality. The assumption on price elasticity of demand is not restrictive in that all the results hold even when this is relaxed; however, this assumption considerably simplifies the exposition and the model. Further, since much of the existing debate focuses on how tax policy affects the locational incentives of firms, it is convenient to assume that total demand for the good is unchanged, and focus on the supply responses.

Production can occur in either state, employing a constant returns to scale production technology with a single non-depreciable input that we call capital.[2] One unit of capital is needed to produce one unit of output. The price of capital in each state is determined by aggregate production in that state (denoted q_i), according to:[3]

$$[1] \quad C(q_i) = \alpha q_i$$

Because the supply of capital is upward-sloping, production is non-separable between states. Our assumption that input prices may not be equalized across states is reasonable if some inputs are immobile; thus, it may be useful to think of land or natural resources as being an important component of capital.

There are many competitive private firms engaged in production. It is useful to distinguish the following types of firms:

- State-x firms: produce in and sell in state x.
- State-y firms: produce in and sell in state y.
- Multijurisdictional firms: produce in state x and sell in state y.

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[1] Differences in demand could arise due to differences in income or population, for example.

[2] Our study examines the apportionment weights placed on production (or input) factors and sales, respectively. In nearly all cases, states weight the labor and property input factors equally. For simplicity, we aggregate all inputs into one, denoted as "capital."

[3] We do not explicitly model the input markets, but only characterize its equilibrium here.

185

We do not consider firms that produce and/or sell in multiple states, or mergers between firms.[4] Production by each type of firm is denoted q_{ij}, with $i, j = \{x,y\}$, where i indexes the state in which production takes place, and j indexes the state into which output is sold. We show below that when these three types of firms exist, there will not be incentives, in general, for firms to produce in state y—the 'high-demand' state—and sell into x; thus, no cross-hauling takes place.

Taxes

Each firm is taxed at rate $\tau < 0.5$ on its accounting income each period[5]—i.e., exclusive of its cost of capital, $rq_{ij}C(q_i)$ (where r denotes the cost of equity capital). Thus, a state-x firm is taxed only by state x, and a state-y firm is taxed only by state y. Those firms each face a tax rate of τ, regardless of the apportionment rule used by the state in which they produce and sell.

A multijurisdictional firm has nexus in both states, so its tax liability depends on the apportionment formula used by state x and state y. State x uses an apportionment formula in which a fraction ω_x of the formula is based on factors associated with production (the location of capital), and the remaining $(1 - \omega_x)$ is based on factors associated with sales, where sales are allocated on the basis of the destination rather than origination. Because multijurisdictional firms produce in state x and sell into state y, they face an effective tax rate of $\tau\omega_x$ in state x. Similarly, state y uses an apportionment formula in which a fraction ω_y of the formula is based on production factors (payroll and property), and the remaining $(1 - \omega_y)$ is based on sales factors; thus, multijurisdictional firms face an effective tax rate of $\tau(1 - \omega_y)$ in state y.

Equilibrium

In any competitive equilibrium, each firm's after tax profits equal zero. This gives us the following zero profit conditions for each type of firm:

$$[2] \quad p_x(1 - \tau) = rC(q_x) = r\alpha(q_{xx} + q_{xy})$$

$$[3] \quad p_y(1 - \tau) = rC(q_y) = r\alpha q_{yy}$$

$$[4] \quad p_y(1 - \tau\omega_x - \tau(1 - \omega_y)) = rC(q_x)$$
$$= r\alpha(q_{xx} + q_{xy})$$

where p_x and p_y are the unit output prices in states x and y. In addition, total output sold in state y equals δ_y in equilibrium:

$$[5] \quad q_{xy} + q_{yy} = \delta_y$$

Finally, since only state-x firms sell output in state x:

$$[6] \quad q_{xx} = \delta_x$$

Solving equations [2]–[6] for the equilibrium prices and quantities gives us:

$$[7] \quad p_x = \frac{\alpha(\delta_x + q_{xy})r}{1 - \tau}$$

$$[8] \quad p_y = \frac{\alpha(\delta_y - q_{xy})r}{1 - \tau}$$

$$[9] \quad q_{xy} = \frac{(\delta_y - \delta_x)(1 - \tau) + \tau\delta_y(\omega_y - \omega_x)}{(2 - 2\tau + \tau(\omega_y - \omega_x))}$$

Remark 1 (No-entry condition): There is no incentive for a firm to produce in state y and sell into state x.

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[4] Thus, we take the boundaries of the firm as given. The results of our model do not necessarily hold if firms producing in different states can costlessly merge. The effects of state taxes on mergers is examined in Gordon and Wilson (1986). However, the analysis in their paper assumes all states use the same apportionment formula.
[5] The restriction $\tau < 0.5$ ensures that the multijurisdictional firm faces a tax rate less than 100 percent.

186

At these prices, there is no incentive for a firm to produce in state y and sell into x as long as:

[10] $\quad \pi_{yx} = p_x(1 - \tau\omega_y - \tau(1 - \omega_x)) - r\alpha q_{yy} \leq 0$

which, substituting for p_x and q_{yy} from equations [7] and [5], and for q_{xy} from equation [9] gives us:

[11] $\quad \dfrac{-\alpha r \tau^2 (\delta_x + \delta_y)(\omega_y - \omega_x)^2}{(1 - \tau)(2 - 2\tau + \tau(\omega_y - \omega_x))} \leq 0$

This no-entry condition is satisfied, for arbitrary ω_x and ω_y, as long as:

[12] $\quad 2 - 2\tau + \tau(\omega_y - \omega_x) \geq 0$

or, $\tau \leq 2/3$.

Welfare

In order to examine the incentives of states in setting formula weights, we first characterize the welfare function for each state, denoted W^X and W^Y, respectively. Each state chooses apportionment weights ω_i to maximize the sum of tax revenues, consumer surplus to residents in that state, and producer surplus received by input owners in that state. These components of the welfare function of each state are described below.

The surplus in consumption accruing to residents in a state i in equilibrium is given by:

[13] $\quad (A_i - p_i)\delta_i$

where A_i denotes the reservation price of buyers in state i, and is assumed large (since demand is inelastic).

Producer surplus in each state is given by:

[14] $\quad \displaystyle\int_0^{q_i} (C(q_i) - C(u))du = \dfrac{\alpha q_i^2}{2}$

where q_i denotes total production in state i. Finally, tax revenues of each state derive both from 'domestic' firms as well as multijurisdictional firms. Thus, for state X, tax revenues are:

[15] $\quad \tau(q_{xx}p_x + q_{xy}p_y\omega_x)$

and for state Y:

[16] $\quad \tau(q_{yy}p_y + q_{xy}p_y(1 - \omega_x))$

where p_x, p_y, and q_{xy} are given according to equations [7]–[9], and q_{xx} and q_{yy} are given according to equations [5] and [6].

Social Optimum

Before explicitly analyzing the incentives for each state to unilaterally alter its formula weights, we first characterize the coordinated solution. Here, the choice of weights is made by a central planner—for example, a federal authority that supersedes the state—that maximizes an aggregate social welfare function, W^T given by:

[17] $\quad W^T = W^X + W^Y$

$\quad = (A^X - p_x)\delta_x + (A^Y - p_y)\delta_y$

$\quad + \dfrac{r\alpha(q_x^2 + q_y^2)}{2} + \tau(q_{xx}p_x + q_{yy}p_y)$

$\quad + q_{xy}p_y(\omega_x + 1 - \omega_y))$

Call the solution to this program the coordinated solution. Note that producer surplus, earned at date 0, is multiplied by the interest rate r to make this comparable with both consumer surplus and tax revenues, which are earned on each future date, in perpetuity.

Proposition 1: In any coordinated solution, $\omega_x^* = \omega_y^*$.

Proof: The first-order condition for ω_x is

[18] $\quad \dfrac{\partial W^T}{\partial \omega_x} = \dfrac{\alpha r(\delta_x + \delta_y)^2(1 - \tau)\tau^2(\omega_y - \omega_x)}{(2 - 2\tau + \tau(\omega_y - \omega_x))^3} = 0$

the solution to which is $\omega_x^* = \omega_y^*$. The same solution is obtained using the first-order condition for ω_y.[6]

Thus, aggregate welfare is maximized when states choose the same formula weights. Because multijurisdictional firms face a combined state tax rate of $\tau(\omega_x + 1 - \omega_y)$, all firms face the same tax rate when states choose the same formula weights. When all firms face a tax rate of τ, total production is equal in each state, which minimizes the cost of satisfying the demand of $(\delta_x + \delta_y)$, hence maximizing equation [17].

Corollary: When $\omega_x = \omega_y$, the multijurisdictional firm is taxed on exactly 100 percent of its income. Moreover, $q_{xy} = (\delta_y - \delta_x)/2$; and so $q_x = q_y = (\delta_x + \delta_y)/2$.

Remark 2. The Multistate Tax Compact may be viewed as a special case of proposition 1, where $\omega_x = \omega_y = 2/3$.

Non-cooperative Equilibrium

In this section we examine the incentives of each state to deviate from the coordinated solution, first by analyzing unilateral deviations and then by characterizing the equilibrium.

First, suppose state x decreases its production factor ω_x, thereby also increasing its sales factor $(1 - \omega_x)$ by the same amount. Multijurisdictional firms now face a combined tax rate less than τ, which induces production to shift from state y to state x. This in turn increases the producer surplus received by owners of capital in state x. Similarly, if state y reduces its production factor ω_y, multijurisdictional firms are now faced with a combined state tax rate

greater than τ, which induces production to shift from state x to state y, thereby increasing the producer surplus received by owners of capital in state y. More generally:

Proposition 2: Producer surplus in each state i unambiguously increases with a decrease in the production factor ω_i in that state. For proof, see Appendix.

The producer surplus effect has generally been the focus of the argument that the primary incentive for states is to increase the sales factor in order to attract business, and increase employment of domestic inputs.[7] However, there are other important incentive effects to consider. A decrease in the production factor by a state will lower consumer surplus in that state (since output prices increase due to production shifting into that state); also, the effects on tax collections in that state are ambiguous. Thus, as states alter their formula weights, the component distributional effects in many cases conflict. Interestingly, however, Proposition 3 shows that states that weight the various components of total surplus equally face unambiguous incentives to deviate from the coordinated solution in particular directions.

Proposition 3: The coordinated solution cannot arise in any non-cooperative equilibrium. In particular: (*i*) State y always decreases ω_y, and (*ii*) for ω_x low enough, state x increases ω_x. For proof, see Appendix.

The intuition behind this result is discussed in detail in the Appendix. Two points are worth emphasizing here. First,

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[6] Since $(\partial^2 W^T / \partial w_x^2 \cdot \partial^2 W^T / \partial w_y^2) - [\partial/\partial w_y \cdot (\partial W^T / \partial w_x)]^2 = 0$, the second-order sufficient condition cannot be used to verify that this is a maximum. However, consider the vector $\vec{v} = \alpha \vec{w}_x + \beta \vec{w}_y$, where α and β are arbitrary, and $\vec{w}_x$ and $\vec{w}_y$ are unit vectors parallel to the ω_x and ω_y axes. Then, $\nabla W^T \bullet \vec{v}$, the directional derivative of W^T in the direction of $\vec{v}$, is equal to zero if and only if $\alpha = \beta$ (i.e., if $\vec{v}$ is parallel to the line $\omega_x = \omega_y$), and is less than zero otherwise. This confirms that all points along the line $\omega_x = \omega_y$ are maxima.

[7] The producer surplus effect is essentially the focus of Goolsbee and Maydew (2000).

the assumption of an upward–sloping supply curve does *not* imply that there are no behavioral responses to changes in apportionment formula by states. As we have seen, an increase in the production factor by state x will shift production out of that state. The key point is that since production does not completely move out, the increase in tax revenues obtained from firms producing in x is large enough to offset the net decline in producer and consumer surplus. Second, the logic behind the state's incentives also suggests why competition among states is not optimal: since state x has an incentive to increase its production factor, production will move out of that state, thus reducing costs there. But, since production costs will increase in the other state, total costs increase as well (this follows from the upward–sloping supply of capital), and state x does not internalize this externality when choosing its apportionment formula.

While we have only considered unilateral deviations so far, the logic behind the competing incentives of the states extends to the case where states simultaneously choose apportionment weights. The Nash equilibrium of this game is characterized in the following proposition.

Proposition 4: The Nash equilibrium is characterized by:

$$\omega_x^* = min\left[\frac{(1-\tau)(\delta_y - \delta_x)}{\tau(2\delta_y - \delta_x)}, 1\right]$$

$$\omega_y^* = 0$$

Proof, see Appendix.

In the non-cooperative equilibrium, importing states will increase their sales factors to 100 percent, whereas exporting states will prefer to increase their production factors. For δ_y sufficiently greater than δ_x, exporting states will choose an interior optimum, where production factors are less than 100 percent. Consequently, the production factors of the importing states will always be lower than that of exporting states.

A SIMPLE TEST

The motive underlying the Uniform Division of Income for Tax Purposes Act (UDITPA), approved in 1957, was to simplify the area of state income taxation, as well as to make it more equitable. The UDITPA was adopted as Article IV of the MTC, developed and approved by a group of state tax administrators, in 1967 (Carlson, Godshaw, and Hyde, 1992). Although compliance with the MTC (including the UDITPA) was voluntary, most of the states adopted an EWF system of apportionment, in accordance with the recommendations (Francis and McGavin, 1992). Subsequently, 29 states deviated from EWF in favor of weighting schemes that put half or more weight on the sales factor, while 13 states still use EWF. This variation in choices of apportionment formula across states provides an opportunity to examine the main prediction of our model: that importing states have incentives to increase their sales factors relative to exporting states.

The four states with no corporate income tax—Nevada, South Dakota, Washington, and Wyoming—are excluded from the analysis. Four other states—California, Colorado, Missouri, and New Mexico—have elective systems that allow firms to choose between EWF and DWSF as well.[8]

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[8] We exclude D.C. from the analysis, since its tax system is dictated by Congress rather than by its government, hence it cannot adjust its system to further its own interests. (For example, Congress, has, historically, prevented D.C. from imposing taxes on non-residents who work there). Mississippi and Louisiana assign DWSF for manufacturers and merchandisers, EWF for all other sectors. Since the components of output we focus on do not fall into these categories, we treat them as using an EWF system.

Table 1 summarizes the weighting schemes used by each state.

Ready data on intra–U.S. trade between states do not exist. Ideally, a measure of net exports for each state i would be given as:

[19] $\quad X_i = Q_i^s - Q_i^d$

where Q_i^d refers to the amount of consumption, and Q_i^s to the amount of production, in each state. To construct a suitable measure of state–level exports and imports, we use various proxies. First, aggregate state income, Y_i, is used as a measure of Q_i^d. Second, since input price variation across states is a key feature of our model, it is sensible to use a measure of the stock of immobile inputs in constructing the supply side measure for each state. While an aggregate land measure is the most natural starting choice, much land may be unusable in production. Hence, we use the sum of total output from agriculture, forestry, fisheries, and mining as an index of usable land, L_i.[9] The measure of net exports we use is given by:

[20] $\quad \hat{X}_i = \ln\left(\dfrac{L_i}{\sum_i L_i}\right) - \ln\left(\dfrac{Y_i}{\sum_i Y_i}\right)$

We use two normalizations here. In order to make the two measures comparable, we use data expressed in *shares* (of national production and demand) rather than *levels*. Next, by taking logarithms, we normalize the measure of exports to reduce the influence of outliers in the estimation procedure. This also implies that a state whose share of usable land equals its share of national income (i.e., $L_i/\sum_i L_i = Y_i/\sum_i Y_i$) is taken to have net exports of 0. Our measure of exports classifies 19 of the 42 states included in the sample as net exporters, and 23 as net importers (see Table 2).

Table 3 shows how the apportionment weights of states vary according to this net exports measure. Six of the seven states which are the largest net importers (and 10 of the top 11) put at least double weight on the sales factor. Conversely, four of the five largest net exporters use equal-weighted formulas. Similarly, 86 percent of the states with net exports lower than

TABLE 1
APPORTIONMENT WEIGHTS BY STATE

Weighting System	State	Frequency
Equal-Weighted Sales Factor	Alabama, Alaska, Delaware, Hawaii, Kansas, Louisiana, Mississippi, Montana, North Dakota, Oklahoma, Rhode Island, Utah, Vermont	13
Double-Weighted Sales Factor	Arizona, Arkansas, Connecticut, Florida, Georgia, Idaho, Illinois, Indiana, Kentucky, Maine, Maryland, New Hampshire, New Jersey, New York, North Carolina, Ohio, Oregon, Pennsylvania, South Carolina, Tennessee, Virginia, West Virginia, Wisconsin	23
Sales Factor > 50%	Iowa, Massachusetts, Michigan, Minnesota, Nebraska, Texas	6
Elective	California, Colorado, Missouri, New Mexico	4
None	Nevada, South Dakota, Washington, Wyoming	4
Total		50

Source: State Tax Guide. 1999. Chicago, Illinois: Commerce Clearing House.
Note: States with elective systems allow firms to choose between EWF and DWSF.

[9] The data are drawn from the Statistical Abstract of the United States (1996), and reproduced in Table 2.

190

TABLE 2
STATE-LEVEL DATA

State	Agriculture (in $ millions)	For. & Fish (in $ millions)	Mining (in $ millions)	Income (in $ billions)	Net Exports
Alabama	343	1,475	1,404	52.5	0.303
Alaska	509	16	8,388	10.1	2.969
Arizona	457	944	1,303	52.1	0.135
Arkansas	255	1,437	366	28.2	0.476
California	5,425	9,024	4,699	537.6	−0.241
Colorado	352	1,245	1,401	54.1	0.201
Connecticut	339	280	63	72.8	−1.577
Delaware	51	195	7	11.5	−0.723
Florida	2,158	3,703	735	212.9	−0.381
Georgia	556	1,949	631	97	−0.338
Hawaii	169	304	25	20.2	−0.609
Idaho	225	1,221	209	13.5	0.995
Illinois	1,010	2,248	1,723	200.9	−0.604
Indiana	393	1,202	753	81.3	−0.451
Iowa	410	3,146	97	40.4	0.690
Kansas	309	1,788	808	38.1	0.520
Kentucky	293	1,752	2,942	47.4	0.842
Louisiana	327	725	12,638	52.4	1.751
Maine	234	222	6	18.3	−0.586
Maryland	470	574	137	92.2	−1.264
Massachusetts	679	313	62	116.5	−1.612
Michigan	625	1,520	1,096	147.8	−0.726
Minnesota	414	2,631	794	71.7	0.166
Mississippi	221	939	704	28.2	0.377
Missouri	436	1,421	371	77.7	−0.458
Montana	106	800	999	10.3	1.406
Nebraska	246	2,977	72	23.9	1.112
Nevada	132	141	2,563	21.5	1.032
New Hampshire	104	101	27	19.6	−1.343
New Jersey	640	387	97	162.9	−1.883
New Mexico	117	551	2,696	18.8	1.372
New York	1,037	1,367	370	349.7	−1.743
North Carolina	608	2,930	422	94.3	−0.077
North Dakota	76	929	752	8.5	1.517
Ohio	748	1,692	1,257	172.1	−0.747
Oklahoma	252	1,348	3,434	41.4	0.986
Oregon	623	1,315	85	42.8	0.042
Pennsylvania	938	1,431	1,844	195.5	−0.744
Rhode Island	122	47	6	16.6	−1.459
South Carolina	275	575	143	46	−0.742
South Dakota	114	1,569	227	9.5	1.453
Tennessee	316	1,099	334	67.7	−0.562
Texas	1,776	4,699	24,930	248.5	1.025
Utah	73	361	1,466	21.2	0.681
Vermont	69	203	27	8.6	−0.266
Virginia	519	1,115	1,163	105.7	−0.539
Washington	1,575	1,902	349	82.2	−0.009
West Virginia	73	195	3,784	21.8	1.411
Wisconsin	479	2,326	136	74.2	−0.134
Wyoming	64	356	3,723	6.7	2.577

Source: (i) income data is for 1990, and is from the *Statistical Abstract of the U.S.*, 1995, U.S. Department of Commerce, Bureau of the Census. (ii) Data on output from agriculture, forestry, fisheries, and mining is for 1990, and is from the U.S. Department of Commerce, *Survey of Current Business*. (iii) All data are expressed in 1987 dollars. (iv) The 'net export' variable is defined in equation [20] in the text.

191

TABLE 3
APPORTIONMENT WEIGHTS VS. EXPORTS

| Export Category | Apportionment Category | | |
	EWF	DWSF	Total
Low	1	6	7
Medium	8	22	30
High	4	1	5
Total	13	29	42

Notes:

1. Each cell denotes the number of states in the given category. "EWF" denotes equal-weighted factor states. "DWSF" states put at least double the weight on the sales factor.

2. 'Net Export' measure is: $\hat{X}_i = \ln(L_i/\Sigma_i L_i) - \ln(Y_i/\Sigma_i Y_i)$, where L_i is state i's output from agriculture, forestry, fisheries, and mining; and Y_i denotes aggregate income of state i.

3. Export categories are defined as follows: states with $\hat{X}_i < -1.25$ are classified as "low" export states, states with $\hat{X}_i < -1.25$ are classified as "high" export states, and others are classified in the "medium" category.

the median put at least double weight on the sales factor, compared with 52 percent of states with net exports greater than the median.[10] All these differences across categories are significant at the 5 percent level. Now, let d_i be a binary variable equal to one if state i has deviated from EWF, and zero otherwise. We then

proceed with maximum likelihood probit estimation, where:

$$d_i = 1 \text{ if } \beta_0 + \beta_1 \hat{X}_i + \varepsilon_i > 0$$

$$d_i = 0 \text{ otherwise}$$

Results are shown in Table 4. β_1 is estimated to be -0.89 (t-statistic of -2.21), and is significant at the five percent level. Together with the estimate of β_0, this result implies that a one percent decrease in a state's share of production relative to its share of exports would increase the probability of its switching to double weighting sales by 19 percent. Moreover, the choice of apportionment weights by states is correctly predicted more than two-thirds of the time by this simple model.[11,12]

One concern with the results relates to potential endogeneity of the "net exports" variable we use. Specifically, since the theory predicts that a state with a low sales factor will tend to drive out manufacturing, this may result in the share of natural resources being disproportionately large. Thus, apportionment weights determine

TABLE 4
APPORTIONMENT WEIGHTS VS. EXPORTS

| Variable | 1990 Data | | 1977 Data | |
	Coefficient	Std. Error	Coefficient	Std. Error
Net exports	−0.8883*	0.3846	−0.7692*	0.3703
Constant	0.9001*	0.3728	0.7174*	0.3555
Number of obs.	42		42	
Log-likelihood	−23.94		−23.44	

Notes:

1. Dependent Variable for state i is 1 if state i's sales factor is at least 0.5, and 0 otherwise. The notes to Table 3 define the 'net exports' measure.

2. * indicates significance at the five percent level.

3. States with no income tax or with an elective weighting system (see Table 1) are excluded.

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[10] In addition, 69 percent of EWF states have positive net exports, compared with 34 percent of DWSF states.

[11] Note that size differences between states are not adequate in explaining the variation in apportionment choices. In fact, we correctly predict that many small states—such as New Hampshire, Maryland, and South Carolina—would double-weight sales, since these are also heavy importers.

[12] An alternative implication of the theory (suggested by a referee) is that states whose share of natural resources (to income) has decreased over time should be more likely to switch to DWSF. Using the 1997 data and 1992 data, we find that of the 21 states whose share has decreased, 17 have switched to DWSF (81 percent); of the 21 states whose share has increased, only 12 have switched (57 percent). A simple probit estimation confirms that this difference is significant at the 9 percent level. Thus, the results based on differences over time in natural resource share are consistent with those based on levels.

192

the measure of net exports, not the other way around. To address this concern, we repeat the estimation with the net exports measure constructed from 1977 data on mining, farming, and agriculture. Since only four states had deviated from EWF prior to 1981, endogeneity should not be a concern. The results of the estimation are similar to the earlier ones (see Table 4, columns 3–4). β_1 is now estimated to be –0.77 (t–statistic of –2.1); moreover, a 1 percent decrease in a state's share of production relative to its share of exports is predicted to increase the probability of its switching to double-weighting sales by 15 percent.[13]

Our theory that states that have retained EWF have done so in an effort to tax immobile capital is also consistent with the decisions by certain states (such as California, Louisiana, and Mississippi) to allow manufacturing firms to use DWSF, while requiring other corporations to use EWF. Goldberg (1998) describes the debate over California's switch to DWSF in this way:

> The general double-weighted sales factor rule was established by SB 1176. At that time, Arco won an amendment that extractive businesses would not use the double-weighted rule because companies 'connected to the land' in both agriculture and extractive businesses are obliged to locate in California in order to conduct their businesses and therefore do not need the double-weighting to do so.

CONCLUSIONS

Based on a two-state equilibrium model of firm location choice, we study the economic incentives of states to select different apportionment formulas. There are three results worth emphasizing in conclusion. First, we show that social welfare is maximized when states coordinate on the choice of apportionment formulas. This result is obtained regardless of which particular formula is chosen. Second, states will in general have unilateral incentives to deviate from any such coordinated solution. Indeed, the coordinated solution will fail to be sustained in any equilibrium where states choose their apportionment formulas noncooperatively. Third, these incentives of states generally conflict. Natural resource importing states have incentives to increase their sales factors, whereas natural resource exporting states will tend to increase their production factors. These predictions concerning the variation in choices of apportionment formulas according to a state's degree of exports and imports is found to be consistent with the observed differences across states in the U.S.

This simple model also sheds some light on two recent debates. First, our model suggests that the simple prediction that all states will have incentives to increase sales factors in a noncooperative setting is generally wrong. The idea underlying this claim is that when states compete over the flow of mobile inputs (such as capital), each will have an incentive not to tax these inputs. Our model instead studies the tax incentives of states when some inputs are immobile. In this setting, natural resource importing states face much the same incentives as before, although the logic underlying this is quite different: they will tend to tax firms that do not produce—but simply sell—in that state. This will shift production towards high-cost domestic firms, with resulting higher output prices for consumers. Being an importing state, the loss to consumers will in general outweigh the gain to domestic producers. However, this is more than offset by the tax revenues gained from

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[13] Another way to correct for endogeneity is to use population rather than income in constructing the net exports measure. The results in this case are even stronger than before ($\beta_1 = -0.92$, t–statistic of –2.21).

foreign firms. Conversely, natural resource exporting states will tend to tax firms selling out of the state. Although this tends to reduce domestic production, the gain in tax revenues from taxing home based exporters more than offsets this loss (as long as the apportionment weight on inputs is not too high). These results hold as long as the supply of inputs is not perfectly elastic.

Second, while our analysis deals with U.S. states, our results help inform the debate regarding whether formula apportionment should be applied internationally. In the international context, countries use separate accounting rather than formula apportionment. This approach leads to contentious disputes between taxpayers and the tax authorities in different countries regarding the proper measurement of income within each political jurisdiction. Our results suggest that a move towards a system of formula apportionment in which formulary weights are equalized across countries is likely to be fragile. Countries would have strong incentives to deviate from any such coordinated system, with net exporters emphasizing production factors—such as property and payroll—and net importers emphasizing output factors, such as sales. Given the relative homogeneity of the states in the U.S. relative to the differences between countries, we expect the extent of deviations in apportionment formulas in an international context to be even more extreme than those observed among U.S. states.

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APPENDIX

Proposition 2

Proof: (i) Consider state x first. As ω_x increases, the change in producer surplus W^{px}, is given by:

$$\frac{\partial W^{px}}{\partial \omega_x} = \alpha(\delta_x + q_{xy}) \cdot \frac{\partial q_{xy}}{\partial \omega_x} = \frac{-\alpha(\delta_x + \delta_y)^2 (1 - \tau)(1 - \tau + \tau(\omega_y - \omega_x))\tau}{(2 - 2\tau + \tau(\omega_y - \omega_x))^3}$$

which is negative because $\tau < 0.5$.

(ii) Now consider state y. As ω_y increases, the change in producer surplus, W^{py}, is given by:

$$\frac{\partial W^{py}}{\partial \omega_y} = -\alpha(\delta_y - q_{xy}) \cdot \frac{\partial q_{xy}}{\partial \omega_y} = -\frac{\alpha(\delta_x + \delta_y)^2 (\tau - 1)^2 \tau}{(2 - 2\tau + \tau(\omega_y - \omega_x))^3}$$

which is negative because $\tau < 0.5$.

Proposition 3

Proof: (i)

$$\left.\frac{\partial W^Y}{\partial \omega_y}\right|_{\omega_y = \omega_x} =$$

$$\left.\frac{r\alpha(\delta_x + \delta_y)\tau(-(\delta_y - \delta_x)(1 - \tau)^2 + \delta_y\tau(\omega_x - \omega_y)(2 - 2\tau - \tau\omega_x) - \delta_x\tau(\omega_x + \omega_y)(1 - \tau))}{(2 - 2\tau - \tau(\omega_x - \omega_y))^3}\right|_{\omega_y = \omega_x}$$

$$= \frac{-r\alpha(\delta_x + \delta_y)\tau((\delta_y - \delta_x)(1 - \tau) + 2\delta_x\tau\omega)}{8(1 - \tau)^2} < 0$$

(ii)

$$\left.\frac{\partial W^Y}{\partial \omega_y}\right|_{\omega_y = \omega_x} =$$

$$\left.\frac{r\alpha(\delta_x + \delta_y)\tau((\delta_y - \delta_x)(1 - \tau)^2 + \delta_y\tau(\omega_y - \omega_x)(2 - 2\tau + \tau\omega_y) - \delta_x\tau(\omega_x + \omega_y)(1 - \tau))}{(2 - 2\tau - \tau(\omega_x - \omega_y))^3}\right|_{\omega_y = \omega_x}$$

$$= \frac{r\alpha(\delta_x + \delta_y)\tau((\delta_y - \delta_x)(1 - \tau)^2 - 2\delta_x\tau\omega)}{8(1 - \tau)^2}$$

which is positive if ω is sufficiently close to zero.

The intuition behind Proposition 3 is instructive in understanding the conflicting incentives of states more generally. To see this, we start by considering the incentives of state y, whose welfare function can be written as:

195

$$[A1] \quad W^Y = (A_y - p_y)\delta_y + r\alpha \frac{(\delta_y - q_{xy})^2}{2} + \tau p_y q_{yy} + (1 - \omega_y)\tau p_y q_{xy}$$

The first two terms represent the surplus to consumers and producers, respectively. The last two terms denote the tax revenues collected from the state-y firm and the multijurisdictional firm. It is convenient to rewrite the tax revenues for state y as:

$$T^Y = \tau p_y \delta_y - \omega_y \tau p_y q_{xy}$$

The first term can be interpreted as the total income tax accruing to state y if $\omega_y = 0$. The second term represents the uncollected tax from the multijurisdictional firms, which arises when $\omega_y > 0$.

In the coordinated solution, $\omega_x = \omega_y$, hence $q_{xy} = (\delta_y - \delta_x)/2$. Consider the effect of a slight increase in ω_y, $\Delta \omega_y$. This reduces state y's sales factor: hence the tax liabilities of the multijurisdictional firms. These firms will therefore tend to increase their production—by an amount Δq_{xy}—which in turn reduces q_{yy} from [5], hence also reduces p_y. For each unit of consumption, the increase in consumer surplus is simply given by the price reduction, $-\Delta p_y$. Hence, the aggregate increase in consumer surplus is given by $-\delta_y \Delta p_y$. With lower prices, and unchanged sales quantities (δ_y), tax revenues from firms fall as well by an amount $\tau \delta_y \Delta p_y$. However, since for every \$1 decrease in prices, tax revenues fall by only \$$\tau$, the increase in consumer surplus more than offsets this decrease in tax revenues, by an amount $-(1 - \tau)\delta_y \Delta p_y$. Using the fact that $\Delta p_y = (-\alpha r/(1 - \tau))\Delta q_{xy}$, this simplifies to $\delta_y \alpha r \Delta q_{xy}$ (see area 1564 in Figure 1). The change in producer surplus (area 1234 in Figure 1), in turn, is given by:

$$\alpha r q_{xy} \Delta q_{yy} = -\alpha r(\delta_y - q_{xy}) \Delta q_{xy}$$

Thus, the net effect—of consumer surplus, producer surplus, and tax revenues from domestic firms—is given by:

$$[A2] \quad \alpha r q_{xy} \Delta q_{xy}$$

which is positive (area 2563 in Figure 1).

It remains to consider what happens to the uncollected tax, $-\omega_y \tau p_y q_{xy}$, as ω_y is increased. In general, there are two effects to consider here: (i) the decrease in state y's *share* of the multijurisdictional firm's tax pie as its sales factor is decreased; (ii) for a given share of this pie, the change in the *size of the tax pie* induced by the multijurisdictional firm's behavioral responses to a change in ω_y. The first effect is negative, since y's share of the multijurisdictional firm's tax pie decreases by an amount $\tau q_{xy} p_y \Delta \omega_y$, as ω_y is increased. Moreover, this effect dominates the first three effects summarized in [A2][14]—see area I in Figure 1. Thus, state y will prefer to decrease ω_y.
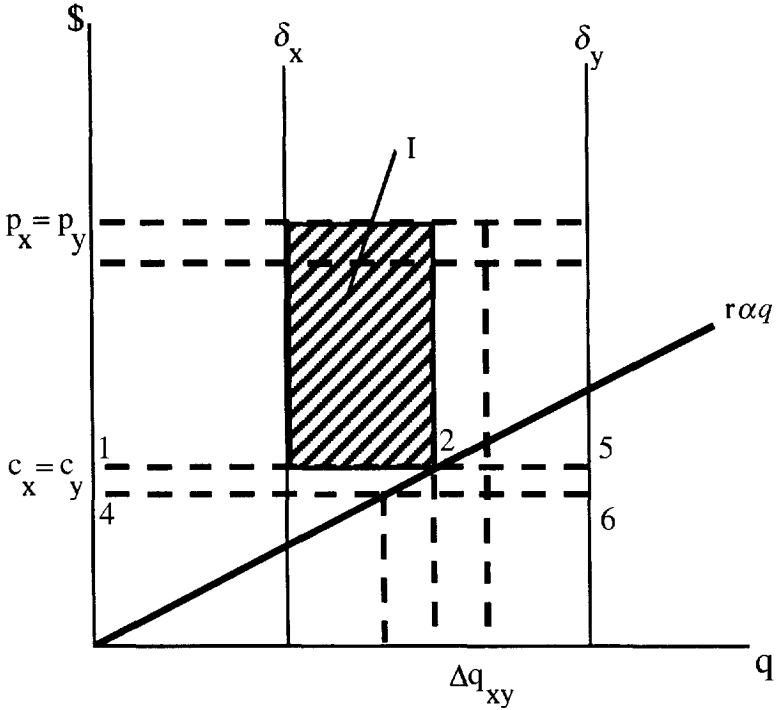
The second effect—namely, the change in the multijurisdictional firm's tax pie caused by changes in q_{xy} and p_y—is also positive.

$$\omega_y \tau(q_{xy} \Delta p_y + p_y \Delta q_{xy}) = \omega_y \tau(q_{xy} \frac{\alpha r}{(1 - \tau)} (-\Delta q_{xy}) + \frac{\alpha r}{(1 - \tau)} (\delta_y - q_{xy}) \cdot \Delta q_{xy} = \omega_y \tau \frac{\alpha r}{(1 - \tau)} \delta_x \Delta q_{xy} > 0$$

In other words, the positive effect of an increase in q_{xy} on the multijurisdictional firm's tax liabilities more than offsets the decrease in tax revenues caused by a decrease in p_y, so that the multijurisdictional firm's tax liabilities increase. For large ω_y, state y's foregone tax revenues are

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[14] $(\tau q_{xy} p_y \Delta \omega_y) - (\alpha r q_{xy} \Delta q_{xy}) = \alpha r \tau(\delta_y^2 - \delta_x^2)/8(1 - \tau) > 0$.

Figure 1. Effects of a Change in ω_y

thus larger; hence, this effect works in the same direction as the sum of the other effects. In summary, state y unambiguously prefers a decrease in w_y.

State x's incentives can be analyzed similarly. Its welfare function is given by:

$$[A3] \quad W^x = (A_x - p_x)\delta_x + \alpha r \frac{(\delta_x + q_{xy})^2}{2} + \tau p_x q_{xx} + \omega_x \tau p_y q_{xy}$$

As before, the first two terms represent consumer and producer surplus, respectively. The last two terms denote tax revenues from domestic firms and from multijurisdictional firms, respectively.
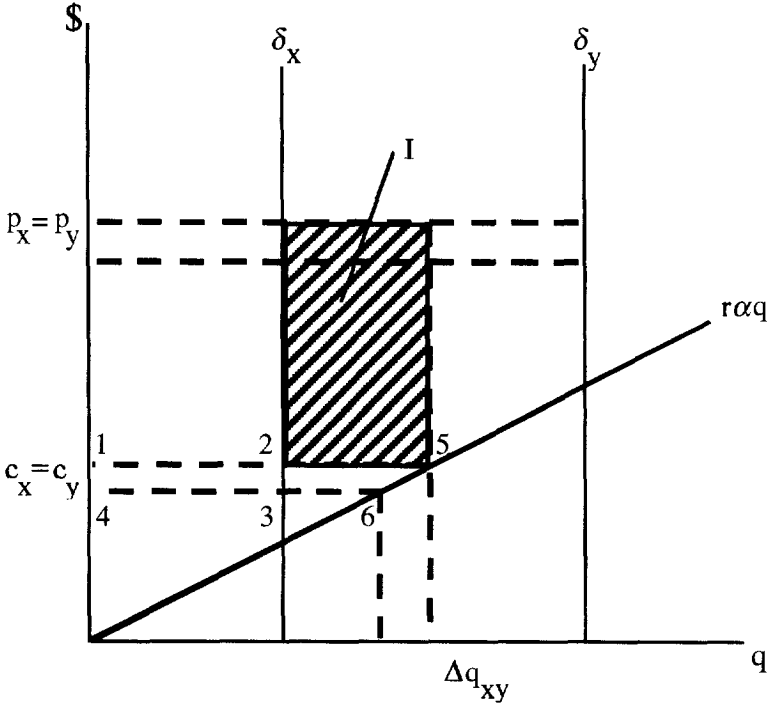
As ω_x is increased by a small amount $\Delta\omega_x$ (from $\omega_x = \omega_y$), the multijurisdictional firm's tax liabilities increase, which will lead such firms to cut back on production, by an amount Δq_{xy}. This results in a decline in aggregate production—hence costs—in state x, which causes output prices in state x to fall as well in equilibrium. Since, for every $1 gain to consumers, tax revenues from domestic firms fall by only τ, the net effect on consumer surplus and tax revenues from domestic firms is positive, and given by $-(1 - \tau)\delta_x \Delta p_x$, which using [7], can be rewritten as $-\alpha r \delta_x \Delta q_{xy}$.

However, since x is an exporting state, this gain is exceeded by the loss in producer surplus, which is given by $\alpha r(\delta_x + q_{xy}) \Delta q_{xy}$. The net welfare effect of the first three terms in [A3] is therefore negative (area 2563 in Figure 2), and given by $\alpha r q_{xy} \Delta q_{xy}$.

Now, as ω_x is increased, state x grabs a larger share of the tax liabilities of the multijurisdictional firm. This increase in revenues, $\tau p_y q_{xy} \Delta \omega_x$, more than offsets the net welfare loss of the first three terms in [A3];[15] see area I in Figure 2. However, as ω_x is increased, the size of this tax pie decreases

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[15] $(\tau p_y q_{xy} \Delta \omega_x) - (\alpha r q_{xy} \Delta q_{xy}) = 3\alpha r \tau(\delta_y^2 - \delta_x^2)/8(1 - t) > 0$.

197

Figure 2. Effects of a Change in ω_x

as well, since the negative effect of a decrease in q_{xy} on the multijurisdictional firm's tax liabilities is greater than the positive effect resulting from an increase in p_y:

$$\omega_x \tau(q_{xy} \Delta p_y + p_y \Delta q_{xy}) = \omega_x \tau \left(q_{xy} \frac{\alpha r}{(1-\tau)} \cdot (-\Delta q_{xy}) + \frac{\alpha r}{(1-\tau)} (\delta_y - q_{xy}) \cdot \Delta q_{xy} \right) = \omega_x \tau \frac{\alpha r}{(1-\tau)} \delta_x \Delta q_{xy} < 0$$

because $\Delta q_{xy} < 0$. As ω_x is increased, this decrease in revenues is larger; this effect tends to offset the positive welfare effects of the other terms. Note that states with low ω_x are harmed less by this adverse effect on existing tax revenues; they will unambiguously prefer an increase in ω_x. For ω_x large enough, however, their preferences may be reversed.

Proposition 4

Proof: Differentiating state x's welfare with respect to ω_x, given state Y has chosen $\omega_y = 0$, yields:

$$[A4] \quad \left. \frac{\partial W^Y}{\partial \omega_y} \right|_{\omega_y = 0} = \frac{\alpha r \tau (1-\tau)(\delta_x + \delta_y)\tau((\delta_y - \delta_x)(1-\tau) - \tau \omega_x (2\delta_y + \delta_x))}{(2 - 2\tau - \tau \omega_x)^2}$$

Setting [A4] equal to zero and solving for ω_x yields:

$$[A5] \quad \omega_x^* = \frac{(1-\tau)(\delta_y - \delta_x)}{\tau(2\delta_y - \delta_x)}$$

198

The second-order condition is satisfied, so [A5] represents the optimal weight on input factors for state X unless $\omega_x^* > 1$, in which case the optimal weight on input factors is its upper bound of one.

Next, differentiating W^Y with respect to ω_y, setting the derivative equal to zero, and solving for ω_y, yields:

[A6] $$\omega_y^* = \frac{(1 - \tau - \tau\omega_x)((\delta_x - \delta_y)(1 - \tau) + \delta_y\tau\omega_x)}{\tau((2\delta_y + \delta_x)(1 - \tau) - \tau\delta_y\omega_x)}$$

There are two cases to consider. First, suppose $\omega_x^* = (1 - \tau)(\delta_y - \delta_x)/\tau(2\delta_y - \delta_x)$. Equation [A6] simplifies to:

[A7] $$\omega_y^* = -\frac{(1 - \tau)\delta_y(\delta_y - \delta_x)^2}{\tau(2\delta_y - \delta_x)(3\delta_y^2 + \delta_y\delta_x - \delta_x^2)} < 0$$

Next, suppose $\omega_x^* = 1$. Equation [A6] simplifies to:

[A8] $$\omega_y^* = \frac{(1 - 2\tau)((\delta_x - \delta_y)(1 - \tau) + \tau\delta_y)}{\tau(\delta_y(2 - 3\tau) + \delta_x(1 - \tau))}$$

Equation [A5] and $\omega_x = 1$ jointly imply that $\tau > (\delta_y - \delta_x)/(3\delta_y - 2\delta_x)$ which in turn implies that [A8] is negative. Therefore, in each case, state Y's welfare is maximized at $\omega_y < 0$; hence, the optimal choice for state Y is to set the weight on input factors equal to zero.

199